Introduction to Mathematical Logic

Winter 2025

Instructor:	Assaf Shani	**Time:**	Tu Th 4:15 - 5:30 pm
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Course materials:

- A Friendly Introduction to Mathematical Logic
  by Christopher C. Leary and Lars Kristiansen.

  The book is freely available at https://milneopentextbooks.org/a-friendly-introduction-to-mathematical-logic/

- Additional notes will be provided on Moodle.

Prerequisites: MATH 222 or MAST 217 or COMP 232 or COEN 231.
Alternatively, familiarity with rigorous mathematical proofs from a more advanced class.

Course Outline: Roughly speaking, mathematical logic studies mathematical objects by first formalizing them in a precise "mathematical language", and then studying how these objects can be defined (or expressed) in this language. We will develop in generality the notion of a "mathematical structure in some language", and formalize notions such as

- *truth* of statements in a structure,
- *provability* of a statement from a set of axioms, and
- *isomorphism* between structures (that is, when two structures are essentially the same).

One basic question is the following: given some axioms (assumptions), is there a structure satisfying such axioms? We will prove Gödel's Completeness Theorem, which asserts that the answer is always yes, as long as there is no "evident logical contradiction between the axioms".

Another fundamental question is whether truth can be axiomatized. That is, can we write a list of axioms from which all mathematical truth can be deduced? Gödel's famous *Incompleteness Theorem* states that this is not possible! For any reasonable list of axioms, there will always be true statements which we cannot prove. Time permitting, we will discuss the formal statement of Gödel's incompleteness theorem and its proof.

Course objectives:

- Understand the following concepts, including examples and relevant results: *mathematical structure*, *isomorphism*, *logical implication*, *formal deduction*, *countable and uncountable sets*, *Peano Arithmetic*, *computable set*, *computable function*.
- Understand the statements, proofs, and applications of the following results: *The Completeness Theorem*, *The Compactness Theorem*, *Cantor's Theorem*, *Gödel's Incompleteness Theorem*.

Assessment: The grade will be based on weekly assignments, a midterm, and a final exam.

Course number: MATH 494-B / MAST 661-B.