Concordia University

EMAT 233 - Final Exam

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Course Examiner: M. Bertola

Date: May 2006.

Time allowed: 3 hours.

[10] Problem 1. Compute the curvature $\kappa(t)$ of the curve $\mathcal{C}$ defined by

$$\vec{r}(t) = t\mathbf{i} + \frac{t^3}{3}\mathbf{j} + \frac{t^2}{2}\mathbf{k}.$$

[10] Problem 2. Find points on the surface $x^2 + 3y^2 + 4z^2 - 2xy = 16$ at which the tangent plane is parallel to the yz–plane.

[10] Problem 3. Find the direction in which the function below increases most rapidly at the indicated point. Find **also** the maximum rate of increase.

$$f(x, y) = \mathrm{e}^{2x} \sin(2y) \ , \qquad P \equiv (0, \pi/8)$$

[10] Problem 4. The D'Alambert equation

$$\frac{\partial^2}{\partial t^2}U(t, x, y) - \frac{\partial^2}{\partial x^2}U(t, x, y) - \frac{\partial^2}{\partial y^2}U(t, x, y) = 0.$$

describes the propagation of small waves on an elastic membrane. Show that the function defined as

$$U(t, x, y) := \cos(ct - ax - by) \ , \qquad c = \sqrt{a^2 + b^2}$$

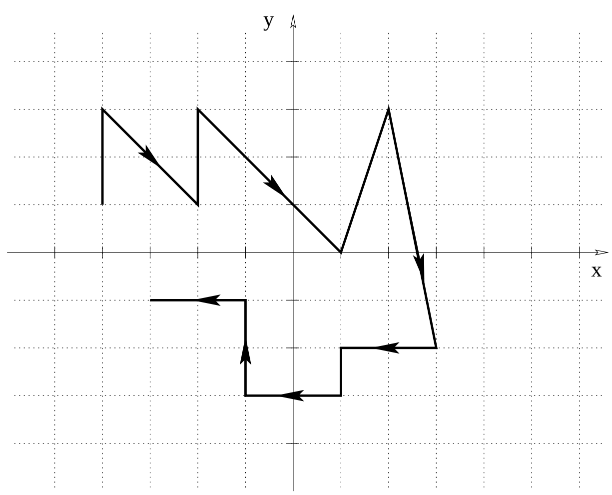
is a solution of the wave equation for any value of the constants a, b (where c is given by the formula written on the right in the equation above).

[10] Problem 5.

Compute the line integral

$$\int_{\mathcal{C}} (x + 2y)\mathrm{d}x + (2x - y)\mathrm{d}y$$

where $\mathcal{C}$ is the contour indicated in figure starting at $(-4, 1)$ and ending at $(-3, -1)$.



[10] Problem 6. Using the appropriate **theorem** (which you must **state**), compute the flux of the curl

$$\iint_S \operatorname{curl}(\vec{F}) \cdot \mathbf{n} \, \mathrm{d}S$$

for the vector–field

$$\vec{F} = y\mathbf{i} - x\mathbf{j} + z\cos(z^3 + \ln(1 + x^2))\mathbf{k}$$

across the upper hemisphere

$$S := \{x^2 + y^2 + z^2 = 1, z \geq 0\}$$

with the normal oriented upwards.

[10] Problem 7. Using the appropriate theorem compute the following line integral in the plane

$$\oint_{\mathcal{C}} 2y \, \mathrm{d}x + 5x \, \mathrm{d}y$$

where $\mathcal{C}$ is the circle $(x-1)^2 + (y+3)^2 = 25$ traversed counterclockwise.

[10] Problem 8. Compute the following double integral by reversing the order of integration

$$\int_0^1 \int_x^1 x^2 \sqrt{1 + y^4} \, \mathrm{d}y \, \mathrm{d}x$$

[10] Problem 9. Using the appropriate theorem (which you must **state**) compute the flux of the vector–field

$$\vec{F}(x, y, z) = (x^2 + 3y + \mathrm{e}^{yz})\mathbf{i} + (3y - x^2)\mathbf{j} + (\ln(1 + x^2 + y^2) + 5z)\mathbf{k}$$

across the surface of the parallelepiped $\{0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 2\}$ with the outwards normal.

[10] Problem 10. Evaluate the following integral by changing it to polar coordinates

$$\int_0^{\sqrt{2}/2} \int_y^{\sqrt{1-y^2}} \frac{y^2}{\sqrt{x^2 + y^2}} \mathrm{d}x \, \mathrm{d}y$$