MAST 699/MAST 833/MATH 494 Algebraic Number Theory
FALL 2021
Assignment 1
Due Friday September 24

Marcus, Chapter 1, exercises 16-28.

Those exercises complete the details of Kummer's proof of case 1 of Fermat's Conjecture given in the first chapter of Marcus (page 4).

The argument can be organized as follows: Let $\xi = \xi_p = e^{2\pi i/p}$ be a p-th root of unity. Assuming that $\mathbb{Z}[\xi]$ is a UFD, it can be show that $x + \xi y$ has the form $u\alpha^p$ for some $\alpha \in \mathbb{Z}[\xi]$ and some unit $u \in \mathbb{Z}[\xi]^\times$. It can then be shown that the equation $x + \xi y = u\alpha^p$, with x, y not divisible by p, implies that $x \equiv y \bmod p$. This is the content of Exercises 16-28 in Chapter 1. The contradiction follows as indicated on page 4 of Marcus.

Some of the exercises were already proven for the case $p = 3$ are are easily extended to the case of a general prime $p > 3$.