

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Pages
Final	December 2018	3
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Special	Only approved calculators are allowed	
Instructions:	Show all your work for full marks.	

MARKS

- [11] **1.** (a) Solve for x (find the **exact** value, do not approximate): $e^{2x} = 2e^x + 24$.
 (b) Let $f(x) = \ln(4-x^2)$ and $g(x) = \sqrt{1-x}$. Find $f \circ g$ and determine its domain.
 (c) Find the inverse function f^{-1} of $f(x) = \log_2(4 + 2^x)$, and determine the domain and the range of $f^{-1}(x)$.

- [8] **2.** Find the limit if it exists, otherwise explain why it does not exist:

Do not use L'Hôpital's Rule

(a) $\lim_{x \rightarrow 2} \frac{|x-2|(x+3)}{x^2+x-6}$ (b) $\lim_{x \rightarrow 1} \frac{x-1}{3-\sqrt{x^2+8}}$

- [5] **3.** Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{\sqrt{16x^2+1}}{x^2-16} \cdot \frac{x^2}{x+4}$$

- [12] **4.** Find the derivatives of the following functions (you don't need to simplify your final answer, but you must show how you calculate it):

(a) $f(x) = x^e e^x + e^2$

(b) $f(x) = \frac{x - \arctan(x)}{1+x^2}$

(c) $f(x) = \cos[x \sin(x) + \sin(x + \cos x)]$

(d) $f(x) = (1+x^2)^{\tan(x)}$ (use logarithmic differentiation)

- [4] **5.** Calculate the second derivative $f''(x)$ of the function

$f(x) = (x+a)(x-a)x^2$ where a is a parameter, and find $f''(1)$.

- [10] 6. Consider the function $y = \sqrt{25 + x}$.
- (a) Use the **definition of derivative** to find the formula for dy/dx .
 - (b) Find the linearization $L(x)$ of the function $y(x)$ at $a = 0$
 - (c) Use this linearization to approximate $\sqrt{30}$.
- [8] 7. Let $f(x) = x^3 - 2x + 3$.
- (a) Find the slope m of the secant line joining the points $(-2, f(-2))$ and $(0, f(0))$.
 - (b) Find all points $x = c$ (if any) on the interval $[-2, 0]$ such that $f'(c) = m$.
- [17] 8. (a) Verify that the point $(2, 1)$ belongs to the curve defined by the equation $x^2 + 2y^2 + 2 = x^3 y^3$, and find an equation of the tangent line to the curve at this point.
- (b) A 13 ft ladder is leaning against a vertical wall of a house when its base starts to slide away (in horizontal direction) from the wall. By the time when the base is at the distance $x = 5$ ft from the wall the base is moving at the rate of $dx/dt = 2$ ft/sec. How fast is the top of the ladder sliding down the wall at that instant?
- (c) Use l'Hôpital's rule to evaluate the $\lim_{x \rightarrow 0} \frac{x \sin(3x)}{(e^{2x} - 1)^2}$.
- [11] 9. (a) Find the absolute maximum and absolute minimum of $f(x) = \frac{x + 2}{x^2 + 5}$ on the interval $[-1, 3]$.
- (b) Find the radius r and the height h of a cylindrical can closed on both top and bottom that has a given volume V , but has the smallest possible surface area A (first express h as a function of r).

[14] **10.** Given the function $f(x) = 2x^3 - 21x^2 + 36x - 9$.

- (a) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and all critical numbers on the x -axis where $f(x)$ has local maximum or local minimum.
- (b) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).
- (c) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Show that the equation $\arctan(x) - \frac{x}{1+x^2} = 0$ has no solution other than $x = 0$. (HINT: use Rolle's Theorem.)