CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	203	All
Examination	**Date**	**Pages**
Final	April 2018	3
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks**	

MARKS

[10] **1.** **(a)** Solve for x : $\log_2(x^2-4) - 2\log_2(x+2) = -2\,.$

(b) Given the function $f = \dfrac{4\cdot 3^x}{6+3^x}$, find the inverse function f^{-1}, and determine the domain and the range of f^{-1}.

[11] **2.** Evaluate the limit if it exists, or explain why the limit does not exist.

(a) $\displaystyle\lim_{x\to 3}\frac{|x-3|}{x^2-x-6}$ **(b)** $\displaystyle\lim_{x\to 1}\frac{x-1}{3-\sqrt{x^2+8}}$ **(c)** $\displaystyle\lim_{x\to\infty}\ln\left(\frac{\sqrt{x^4+9x^6}}{(2+3x)(4+x^2)}\right)$

[5] **3.** Calculate the second derivative $f''(x)$ of the function $f(x) = x^{1/2}(\sqrt{x} - x^{-1/2})\, e^{a\,x}$ where a is a parameter, and find $f''(0)$.

[16] **4.** Find the derivatives of the following functions (**show your work for full marks**):

(**a**) $f(x) = 2^{x+1}\cdot(x^3+3\,x^{1/3})$

(**b**) $f(x) = \ln\left(\dfrac{x^4}{x+3}\right) + e^3$

(**c**) $f(x) = \dfrac{\arctan(x)}{\tan(x)+x}$

(**d**) $f(x) = \ln[x^2\sin(x) + x\cos(x^2)]$

(**e**) $f(x) = (1+2x)^{x^2}$ (use logarithmic differentiation)

[15] **5.** **(a)** Verify that the point (2,1) belongs to the curve defined by the equation $xy + 2\sqrt{3+y^2} = x^3 - 2$, and find the equation of the tangent line to the curve at this point.

(b) A spherical snowball is melting in such a way that its diameter D is decreasing at the rate of $dD/dt = -0.1\,\text{cm/min}$. At what rate the volume V of the snowball is decreasing when the diameter is $9\,\text{cm}$?
(NOTE: the volume of a sphere with radius r is $V = 4\pi r^3/3$)

(c) Use the l'Hôpital's rule to evaluate the $\lim\limits_{x\to 0} \dfrac{e^{x^2}-1}{x\sin(2x)}$.

[6] **6.** Let $f(x) = 3 + x + 3\,x^2 - x^3$.

(a) Find the slope m of the secant line joining the points $(0, f(0))$ and $(3, f(3))$.

(b) Find all points $x = c$ (if any) on the interval (0,3) such that $f'(c) = m$.

[9] **7.** Consider the function $f(x) = \sqrt{2x+1}$.

(a) Use the **definition of the derivative** to find the formula for $f'(x)$.

(b) Write the linearization formula for f at $a = 4$.

(c) Use this linearization to approximate the value of $f(5) = \sqrt{11}$.

[12] **8.** **(a)** Find the absolute extrema of $f(x) = \dfrac{2x}{x^2+x+1}$ on the interval $[0, 3]$.

(b) A rectangle is inscribed with its base on the x-axis and its upper corners on the parabola $y = 12 - x^2$. Find the dimensions of such rectangle with the maximum possible area.

[16] **9.** Given the function $f(x) = x\,e^{-2x^2}$.

(a) Find the domain of f and check for symmetry. Find asymptotes of f (if any).

(b) Calculate $f'(x)$ and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).

(d) Sketch the graph of the function $f(x)$ using the information obtained above.

[5] **Bonus Question.** Let $y = f(x)$ and $u = g(x)$ be twice differentiable functions. Use the Chain rule to derive the following formula for the second derivative of the composite function $h(x) = f(g(x))$:

$$h''(x) = f''(u)\,(g'(x))^2 + f'(u)\,g''(x)$$