CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All

Examination	Date	Duration
Midterm Test	23 October, 2016	1 h 30 min

Special Instructions:	Only approved calculators are allowed
	Show your work for full marks

1. (12 marks): **(a)** Solve for x (find the *exact* values, do not approximate):

$\quad$ **(i)** $27 = 3^{5x}\, 9^{x^2}$ $\qquad$ **(ii)** $\ln(\ln x) = 0$

$\quad$ **(b)** Let $f(x) = \sqrt{9 - 3x}$ and $g(x) = 3 - x^2$. Find the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

$\quad$ **(c)** Let $f(x) = 2^{-x} - 1$. Find the inverse function f^{-1} and the domain and range of f^{-1}.

2. (8 marks) $\quad$ Find the limit or explain why the limit does not exist:

$\quad$ **(a)** $\displaystyle \lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

$\quad$ **(b)** $\displaystyle \lim_{x \to -5} \frac{x^2 - 25}{|x+5|}$

3. (6 marks) $\quad$ Find **(a)** all horizontal and **(b)** all vertical asymptotes of the graph

$$y = \frac{\sqrt{4x^6 + 2x^2 + 1}}{x^3 + x^2 - 6x}$$

4. (4 marks) $\quad$ Consider the following piecewise-defined function:

$$f(x) = \begin{cases} ax + 2b & \text{if} \quad x \le 0 \\ x^2 + 3a - b & \text{if} \quad 0 < x \le 2 \\ 3x - 5 & \text{if} \quad x > 2 \end{cases}$$

$\quad$ For what values of a and b is the function continuous at every x? Explain.

(continued on the other side)

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5. (12 marks)   Find the derivatives of the following functions. (you don't need
to simplify the final answer, but you must show how you calculate it):

(a) $f(x) = x\sqrt{x}\,(x + x^{-3/2})$

(b) $f(x) = \dfrac{x3^x}{(x + 3^{-x})}$

(c) $f(x) = \cos\left(\sqrt[3]{x}\right) + \sqrt[3]{\tan x}$

(d) $f(x) = \sin[\,x\,\cos(x) + x^2 + \cos^2(x)\,]$

6. (8 marks)   Given the function $f(x) = \sqrt{3x + 7}$,

(a)   Calculate $f'(x)$ using its definition as a limit of difference quotient.

(b)   Check that your calculation is correct using standard differentiation rules.

(c)   Write equation of the tangent line to the curve $y = f(x)$ at the
point $(3, 4)$.

Bonus Question (3 marks).   Find a formula for the n-th derivative $f^{(n)}(x)$ if $f(x) = \dfrac{1}{1 - x}$
and n is any positive integer.