MATH369 Fall 2020
Assignment 1
Due Friday September 25

Abstract Algebra by Dummit and Foote, Section 0.1, 0.2 and 0.3 and 1.1

1. Consider the relation $\sim$ on the cartesian plane $\mathbb{R}^2 = \{(x, y) \ : \ x, y \in \mathbb{R}\}$ given by:

$$(x_1, y_1) \sim (x_2, y_2) \iff y_1 = y_2.$$

   (a) Prove that $\sim$ is an equivalence relation.

   (b) Describe the equivalence classes and give a complete set of equivalence classes representatives.

2. Consider the relation $\sim$ on the cartesian plane $\mathbb{R}^2 = \{(x, y) \ : \ x, y \in \mathbb{R}\}$ given by:

$$(x_1, y_1) \sim (x_2, y_2) \iff \text{either } y_1 = y_2 \text{ or } x_1 = x_2.$$

   Explain why this is not an equivalence relation.

3. (a) Show that the square of every odd integer is congruent to 1 modulo 8.

   (b) Prove that for any integers a and b, $a^2 + b^2$ is never congruent to 3 or 7 modulo 8.

4. (a) Find the order of each element in the additive group $\mathbb{Z}/15\mathbb{Z}$.

   (b) Write down the multiplication table in the multiplicative group $(\mathbb{Z}/15\mathbb{Z})^*$, and find the order of each element.