

# Periodic Islands for 2-dim Maps

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# Contents

Periodic Islands for 2-dim  
Maps

## Contents

Periodic Islands for 2-dim  
Maps

"Invariant measure" and  
map  $G$

Example:  $f$  is the logistic  
map

References

Periodic Islands for 2-dim Maps

"Invariant measure" and map  $G$

Example:  $f$  is the logistic map

Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a map. We want to consider a process, which is not a map, and represents situation when  $f$  on each step uses not only current information but also some information from the past. For current state  $x_n$  and  $0 < \alpha < 1$  we define:

$$x_{n+1} = T(x_n) = f(\alpha x_n + (1 - \alpha)x_{n-1}).$$

# "Invariant measure" and map $G$

Periodic Islands for 2-dim  
Maps

Contents

Periodic Islands for 2-dim  
Maps

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map  $G$

Example:  $f$  is the logistic  
map

References

We are interested in something we could call an "invariant measure" of the process. We consider ergodic averages

$$A_g(x_0, x_{-1}) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} g(x_i).$$

They are related to ergodic averages of the map  $G: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined by

$$G(x, y) = (y, f(\alpha y + (1 - \alpha)x)).$$

# Example: $f$ is the tent map

Periodic Islands for 2-dim  
Maps

Contents

Periodic Islands for 2-dim  
Maps

"Invariant measure" and  
map  $G$

Example:  $f$  is the logistic  
map

References

We considered the example where  $f : [0, 1] \rightarrow [0, 1]$  is the logistic map

$$f(x) = 4 \cdot x \cdot (1 - x).$$

# The parameters we consider

$\alpha = 0.3235$  10 clusters

one iteration of  $G$  moving by 3 positions clockwise

one iteration of  $G^3$  moving by 1 position counter-clockwise

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$\alpha = 0.2785$  7 clusters

one iteration of  $G$  moving by 2 positions clockwise

one iteration of  $G^3$  moving by 1 position counter-clockwise

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$\alpha = 0.1278, 0.1284, 0.1286$  : 5 clusters

iteration shown in the figure

Contents

Periodic Islands for 2-dim  
Maps

"Invariant measure" and  
map  $G$

Example:  $f$  is the logistic  
map

References



Paweł Góra, Abraham Boyarsky, Zhenyang Li and Harald Proppe, *Statistical and Deterministic Dynamics of Maps with Memory*, Discrete and Continuous Dynamical System - A, **37** (8) (2017), 4347–4378, DOI: 10.3934/dcds.2017186, preprint, <http://arxiv.org/abs/1604.06991>.



Paweł Góra, Abraham Boyarsky and Zhenyang Li, *Singular SRB measures for a non  $1-1$  map of the unit square*, Journal of Stat. Physics **165** (2016), 409–433, DOI: 10.1007/s10955-016-1620-y, available at <http://arxiv.org/abs/1607.01658>, full-text view-only version: <http://rdcu.be/kod0>