```
> with(plots):Digits:=100:interface(displayprecision=10):with
  (linalg):


>
  N:=3;

  bb:=vector(N+1,[]):#for printing only = b[]
  beta:=vector(N,[]):

  alpha:=vector(N,[]):
  gamm=vector(N,[]):#heiths of lower ends of hanging branches
                        # alpha[i]+gamma[i]<1   !!!!!!!
                        #if gamma[i]>0

  alpha[1]:=0.5:beta[1]:=2:gamm[1]:=0:

  alpha[2]:=1:beta[2]:=2:gamm[2]:=0:#
  #alpha[3]:=0.5:beta[3]:=2:gamm[3]:=0.0:  #
  #alpha[4]:=0.4:beta[4]:=5:gamm[4]:=0.4:
  #alpha[5]:=1.0:beta[5]:=9:gamm[5]:=0:
  #alpha[6]:=0.6:beta[6]:=7:gamm[6]:=0.2:#
  #alpha[7]:=1.0:beta[7]:=6:gamm[7]:=0.0:  #
  alpha[N]:=0.5:                gamm[N]:=0.5:
  i:='i':beta[N]:=alpha[N]/(1-sum(alpha[i]/beta[i],i=1..N-1));
  if beta[N] < 0 then print(`ERROR - make beta's larger `) fi;

  print(`alpha =`,alpha);
  print(`beta =`,beta);
  print(`gamma =`,gamm);
  i:='i':
  beta_const:=sum(alpha[i],i=1..N);
  i:='i':
  #for j from 1 to N do
  #beta[j]:=beta_const;
  #od:


  b[1]:=0:
  for j from 1 to N do
  b[j+1]:=b[j]+alpha[j]/beta[j]:
   od: i:='i':
  b[N+1]:=1:
  ag:=vector(N,[]):
  al:=vector(N,[]):
  a:=vector(N,[]):
  c:=vector(N,[]):
```

```
   for j from 1 to N do
   bb[j]:=b[j];
   ag[j]:=beta[j]*b[j];
   al[j]:=-1+beta[j]*b[j+1];
   od:
   bb[N+1]:=1:
   for j from 1 to N do
   a[j]:=ag[j]-gamm[j];
    od:

   print(`b =`,bb);
   print(`ag =`,ag);
   print(`al =`,al);
   print(`a =`,a);
   print(`gamma =`,gamm);
>
>
> # ag shows maximal digit (greedy)
   #  al shows minimal digit (lazy)  #### if ag[j]=al[j] then  j is
   onto branch and there is
                                                                  #
   no choice there
   # a shows digits assigned automatically using the vector U: U(j)
   =1 lazy
   #                                                          U(j)=
   0 greedy
   # we can assign digit a arbitrarily between minimum and maximum
   and then put 2 into vector U



   # Now we will name points c[i] (there is KK + number of 2's in U
   points c[i])
   # and create a vectors sidec[], ineqc[],signc[] which shows the
   character of the point c[i]
   Kc:=0:# new number of c points
   for j from 1 to N do if alpha[j]<1 then Kc:=Kc+1 fi od:
   for j from 1 to N do if (gamm[j]>0 and alpha[j]+gamm[j]<1) then
   Kc:=Kc+1 fi od:
   print(`Kc =`,Kc);
   c:=vector(2*N,[]):
   sidec:=vector(2*N,[]):# 1 left (use uT), 0 right (use T)
   j_of_c:=vector(2*N,[]):# shows the index of the interval
   associated with c

   cj:=1:# this is the new index for c points
   for j from 1 to N do
```

```
   if (alpha[j]<1 and gamm[j]=0) then   c[cj]:=b[j+1]; sidec[cj]:=
   0;j_of_c[cj]:=j;cj:=cj+1 fi;
   if (alpha[j]<1 and gamm[j]+alpha[j]=1) then  c[cj]:=b[j]; sidec
   [cj]:=1;j_of_c[cj]:=j;cj:=cj+1 fi;
   if (gamm[j]>0 and gamm[j]+alpha[j]<1) then  c[cj]:=b[j]; sidec
   [cj]:=1;j_of_c[cj]:=j;cj:=cj+1 ;
                        c[cj]:=b[j+1]; sidec[cj]:=0;j_of_c[cj]:=j;
   cj:=cj+1 fi;




   od:
   print(`c =`,c);
   print(`sidec =`,sidec);
   print(`j_of_c =`,j_of_c);


>
>


>
   int_of_x:=x->piecewise(x<b[2],1,# This function needs additions
   by hand for
                                    # N>9 . Automathic procedure
   causes plotting problems
                                    # but is used in other
   programs
                            x<b[3],2,
                            x<b[4],3,
                            x<b[5],4,
                            x<b[6],5,
                            x<b[7],6,
                            x<b[8],7,
                            x<b[9],8,
                                 9);
   int_of_x:=x->piecewise(x<=b[2],1,# This function needs additions
   by hand for
                                    # N>9 . Automathic procedure
   causes plotting problems
                                    # but is used in other
   programs
                            x<=b[3],2,
                            x<=b[4],3,
                            x<=b[5],4,
                            x<=b[6],5,
```

```
                                x<=b[ 7] , 6,
                                x<=b[ 8] , 7,
                                x<=b[ 9] , 8,
                                      9) ;
x: =' x' :
uT: =x- >bet a[ ui nt _of _x( x) ] * x- a[ ui nt _of _x( x) ] ;
T: =x- >bet a[ i nt _of _x( x) ] * x- a[ i nt _of _x( x) ] ;
Tc: =vect or ( Kc+2, [ ] ) :
f or  j  f r om 1  t o  Kc  do
i f  si dec[ j ] =0  t hen    Tc[ j ] : =T( c[ j ] ) ;
                el se   Tc[ j ] : =uT( c[ j ] ) f i ;
od:
Tc[ 1] : =0. 500000;
Tc[ 2] : =0. 50000000;
pr i nt ( ` Tc  =  ` ,  Tc) ;

#pl ot ( [ ' uT( x) ' , x, 0, 1, Tc[ 1] , Tc[ 2] , Tc[ 3] ] , x=0. . 1, t hi ckness=[ 2, 1, 1,
1, 1, 1, 1] ) ;
pl ot ( [ ' T( x) ' , x, 0, 1, Tc[ 1] , Tc[ 2] , Tc[ 3] ] , x=0. . 1, t hi ckness=[ 2, 1, 1, 1,
1, 1, 1, 1] ) ;
```

$$N := 3$$

$$\beta_3 := 2.0000000000$$

$$alpha =, \left[\begin{array}{ccc} 0.5000000000 & 1 & 0.5000000000 \end{array} \right]$$

$$beta =, \left[\begin{array}{ccc} 2 & 2 & 2.0000000000 \end{array} \right]$$

$$gamma =, \left[\begin{array}{ccc} 0 & 0 & 0.5000000000 \end{array} \right]$$

$$\beta const := 2.0000000000$$

$$b =, \left[\begin{array}{cccc} 0 & 0.2500000000 & 0.7500000000 & 1 \end{array} \right]$$

$$ag =, \left[\begin{array}{ccc} 0 & 0.5000000000 & 1.5000000000 \end{array} \right]$$

$$al =, \left[\begin{array}{ccc} -0.5000000000 & 0.5000000000 & 1.0000000000 \end{array} \right]$$

$$a =, \left[\begin{array}{ccc} 0 & 0.5000000000 & 1.0000000000 \end{array} \right]$$

$$gamma =, \left[\begin{array}{ccc} 0 & 0 & 0.5000000000 \end{array} \right]$$

$$Kc =, 2$$

$$c =, \left[\begin{array}{cccccc} 0.2500000000 & 0.7500000000 & c_3 & c_4 & c_5 & c_6 \end{array} \right]$$

$$sidec =, \left[\begin{array}{cccccc} 0 & 1 & sidec_3 & sidec_4 & sidec_5 & sidec_6 \end{array} \right]$$

$$j\_of\_c =, \left[\begin{array}{cccccc} 1 & 3 & j\_of\_c_3 & j\_of\_c_4 & j\_of\_c_5 & j\_of\_c_6 \end{array} \right]$$

$$uint\_of\_x := x \to piecewise\left(x < b_2, 1, x < b_3, 2, x < b_4, 3, x < b_5, 4, x < b_6, 5, x < b_7, 6, x < b_8, 7, x < b_9, 8, 9\right)$$

$$int\_of\_x := x \to piecewise\left(x \le b_2, 1, x \le b_3, 2, x \le b_4, 3, x \le b_5, 4, x \le b_6, 5, x \le b_7, 6, x \le b_8, 7, x \le b_9, 8, 9\right)$$

$$uT := x \to \beta_{uint\_of\_x(x)}\, x - a_{uint\_of\_x(x)}$$

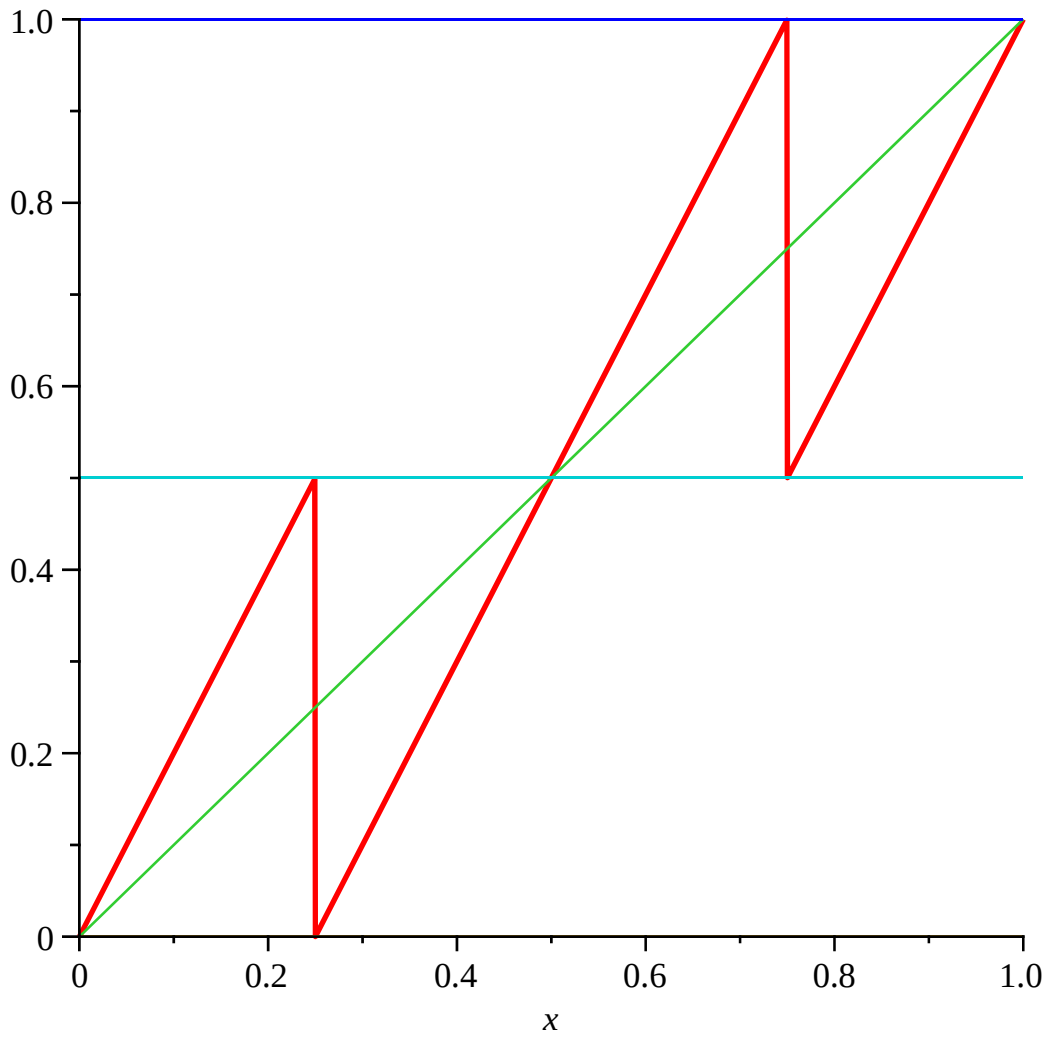
$$T := x \to \beta_{int\_of\_x(x)}\, x - a_{int\_of\_x(x)}$$

$$Tc_1 := 0.5000000000$$

$$Tc_2 := 0.5000000000$$

$$Tc = , \left[\, 0.5000000000 \quad 0.5000000000 \quad Tc_3 \quad Tc_4 \,\right]$$

```
Warning, unable to evaluate 1 of the 7 functions to numeric
values in the region; see the plotting command's help page to
ensure the calling sequence is correct
```




```
>
> delta1:=(xw,yw)->piecewise(xw<=yw,0,1):
  delta2:=(xw,yw)->piecewise(xw<yw,0,1):
  #symbolic versions of alpha, beta
  alps:=vector(N,[]):
```

```
bets:=vector(N,[]):
# Procedure producing coefficients in the non simplified
equations
Dc:=proc(k,x) local val,ii4;

if (sidec[k]=0 and k<= Kc) then
            val:=S[k];
           val:=val-sum(delta1(x,Tc[ii4])*SS[k,ii4]*delta1(0.5,
sidec[ii4])/bets[j_of_c[ii4]],ii4=1..Kc);
           val:=val-sum(delta1(Tc[ii4],x)*SS[k,ii4]*delta1(sidec
[ii4],0.5)/bets[j_of_c[ii4]],ii4=1..Kc);
           val:=val-delta2(Tc[k],x)/bets[j_of_c[k]];
end if;
if (sidec[k]=1 and k<= Kc) then
            val:=S[k];
           val:=val-sum(delta1(x,Tc[ii4])*SS[k,ii4]*delta1(0.5,
sidec[ii4])/bets[j_of_c[ii4]],ii4=1..Kc);
           val:=val-sum(delta1(Tc[ii4],x)*SS[k,ii4]*delta1(sidec
[ii4],0.5)/bets[j_of_c[ii4]],ii4=1..Kc);
           val:=val-delta2(x,Tc[k])/bets[j_of_c[k]];
end if;
if ( k = Kc+1) then
            val:=1-sum(1/bets[ii4],ii4=1..N);
           val:=val+sum(delta1(x,Tc[ii4])*delta1(0.5,sidec[ii4])
/bets[j_of_c[ii4]],ii4=1..Kc);
           val:=val+sum(delta1(Tc[ii4],x)*delta1(sidec[ii4],0.5)
/bets[j_of_c[ii4]],ii4=1..Kc);

end if;
           return val;
      end proc;
for k from 1 to Kc+1 do
Dc(k,0.6);
od;
#
```

```
Dc := proc(k, x)
    local val, ii4;
    if sidec[k] = 0 and k <= Kc then
        val := S[k];
        val := val - (sum(delta1(x, Tc[ii4]) * SS[k, ii4] * delta1(0.5000000000, sidec[ii4])
        / bets[j_of_c[ii4]], ii4 = 1 .. Kc));
        val := val - (sum(delta1(Tc[ii4], x) * SS[k, ii4] * delta1(sidec[ii4], 0.5000000000)
        / bets[j_of_c[ii4]], ii4 = 1 .. Kc));
        val := val - delta2(Tc[k], x) / bets[j_of_c[k]]
    end if;
```

```
    if sidec[k] = 1 and k <= Kc then
        val := S[k];
        val := val - (sum(delta1(x, Tc[ii4]) * SS[k, ii4] * delta1(0.5000000000, sidec[ii4])
        / bets[j_of_c[ii4]], ii4 = 1 ..Kc) );
        val := val - (sum(delta1(Tc[ii4], x) * SS[k, ii4] * delta1(sidec[ii4], 0.5000000000)
        / bets[j_of_c[ii4]], ii4 = 1 ..Kc) );
        val := val - delta2(x, Tc[k]) / bets[j_of_c[k]]
    end if;
    if k = Kc + 1 then
        val := 1 - (sum(1 / bets[ii4], ii4 = 1 ..N) );
        val := val + sum(delta1(x, Tc[ii4]) * delta1(0.5000000000, sidec[ii4]) / bets[j_of_c[ii4
        ]], ii4 = 1 ..Kc);
        val := val + sum(delta1(Tc[ii4], x) * delta1(sidec[ii4], 0.5000000000) / bets[j_of_c[ii4
        ]], ii4 = 1 ..Kc)
    end if;
    return val
end proc
```

$$S_1 - \frac{SS_{1,1}}{bets_1}$$

$$S_2 - \frac{SS_{2,1}}{bets_1} - \frac{1}{bets_3}$$

$$1.0000000000 - \frac{1}{bets_2} - \frac{1}{bets_3} \qquad \textbf{(1)}$$

```
> # Chosing x's and actually producing the coefficients in the non
  simplified equations ES
  CofD:=matrix(Kc+1, Kc+1, []):
  xc:=vector(Kc+1, []):
  Tc[Kc+1]:=0;
  Tc[Kc+2]:=1;
  Tcl:=convert(Tc, list):
  Tc_s:=sort(Tcl, `<`):
  print(Tc_s);

  for i from 1 to Kc+1 do
  x:=(Tc_s[i]+Tc_s[i+1])/2;
  xc[i]:=x;
  for k from 1 to Kc+1 do
  CofD[i, k]:=Dc(k, x);
  od:
  od:
  print(` x's = `, xc);
  print((CofD));
```

>

$$Tc_3 := 0$$

$$Tc_4 := 1$$

$$[0, 0.5000000000, 0.5000000000, 1]$$

$$x's = , \left[\begin{array}{ccc} 0.2500000000 & 0.5000000000 & 0.7500000000 \end{array}\right]$$

$$\left[\begin{array}{ccc} S_1 - \frac{SS_{1,2}}{bets_3} - \frac{1}{bets_1} & S_2 - \frac{SS_{2,2}}{bets_3} & 1.0000000000 - \frac{1}{bets_1} - \frac{1}{bets_2} \\ S_1 - \frac{1}{bets_1} & S_2 - \frac{1}{bets_3} & 1.0000000000 - \frac{1}{bets_1} - \frac{1}{bets_2} - \frac{1}{bets_3} \\ S_1 - \frac{SS_{1,1}}{bets_1} & S_2 - \frac{SS_{2,1}}{bets_1} - \frac{1}{bets_3} & 1.0000000000 - \frac{1}{bets_2} - \frac{1}{bets_3} \end{array}\right] \quad (2)$$

```
> #  producing the coefficients in the  simplified equations EQS
  # finding where Tc[i] is (between which xc[i]'s)

  ixc:=vector(Kc,[]):
  for i from 1 to Kc do
  level:=Tc[i];
  for j from 1 to Kc+1 do
  if (sidec[i]=1 and xc[j]<level) then ixc[i]:=j fi;
  if (sidec[i]=0 and xc[j]<=level) then ixc[i]:=j fi;
  od:
  od:
  print(`ixc = `,ixc);

  for i from 1 to Kc+1 do
  vD[i]:=row(CofD,i);
  od:
  for i from 1 to Kc do
  if sidec[i]=0 then nvD[i]:=evalm(vD[ixc[i]+1]-vD[ixc[i]]) fi;
  if sidec[i]=1 then nvD[i]:=evalm(vD[ixc[i]]-vD[ixc[i]+1]) fi;
  od:



  nvD[Kc+1]:=evalm(vD[2]);
  CofD_n:=matrix(Kc+1,Kc+1,[]):
  for i from 1 to Kc+1 do
  for j from 1 to Kc+1 do
  CofD_n[i,j]:=nvD[i][j];
  od;od;
  evalm(CofD_n);
```

$$ixc = , \left[\begin{array}{cc} 2 & 1 \end{array} \right]$$

$$nvD_3 := \left[\begin{array}{ccc} S_1 - \frac{1}{bets_1} & S_2 - \frac{1}{bets_3} & 1.0000000000 - \frac{1}{bets_1} - \frac{1}{bets_2} - \frac{1}{bets_3} \end{array} \right]$$

$$\left[\begin{array}{ccc} -\frac{SS_{1,1}}{bets_1} + \frac{1}{bets_1} & -\frac{SS_{2,1}}{bets_1} & \frac{1}{bets_1} \\ -\frac{SS_{1,2}}{bets_3} & -\frac{SS_{2,2}}{bets_3} + \frac{1}{bets_3} & \frac{1}{bets_3} \\ S_1 - \frac{1}{bets_1} & S_2 - \frac{1}{bets_3} & 1.0000000000 - \frac{1}{bets_1} - \frac{1}{bets_2} - \frac{1}{bets_3} \end{array} \right] \quad (3)$$

>

```
> for i from 1 to Kc+1 do # this part works only after runnig the
  program to the end and returning
  vD[i]:=row(CofD_n,i);            # Then, RUN IT TWICE
  od;
  for i from 1 to Kc do
  if sidec[i]=0 then
                  eta[i]:=1-alpha[j_of_c[i]]-gamm[j_of_c[i]];
                      else
                  eta[i]:=gamm[j_of_c[i]];
  end if;
  print(i,eta[i]);
  od;
  for i from 1 to Kc+1 do
  sD[i]:=eta[1]*vD[1][i]+eta[2]*vD[2][i]+vD[Kc+1][i];# This line
  may need to be changed by hand
  od;
  for i from 1 to N do
  bets[i]:=beta[i];
  alps[i]:=alpha[i];
  od:

  det(CofD_n);
```

>
>
>

$$vD_1 := \left[\begin{array}{cccc} -\frac{SS_{1,1}}{bets_1} + \frac{1}{bets_1} & -\frac{SS_{2,1}}{bets_1} & \frac{1}{bets_1} \end{array} \right]$$

$$vD_2 := \left[-\frac{SS_{1,2}}{bets_3} \quad -\frac{SS_{2,2}}{bets_3} + \frac{1}{bets_3} \quad \frac{1}{bets_3} \right]$$

$$vD_3 := \left[S_1 - \frac{1}{bets_1} \quad S_2 - \frac{1}{bets_3} \quad 1.0000000000 - \frac{1}{bets_1} - \frac{1}{bets_2} - \frac{1}{bets_3} \right]$$

1, 0.5000000000

2, 0.5000000000

$$sD_1 := -2.220446049\ 10^{-16}$$

$$sD_2 := -2.220446049\ 10^{-16}$$

$$sD_3 := 0.0000000000$$

$$9.860761315\ 10^{-32} \qquad \textbf{(4)}$$

```
> 
> 
> ud:=vector(50):Digits:=100;NN:=50;#Expansion with variable
  slopes
  d:=vector(50):
  xx:=evalf(rand()/10^12);
  xxt:=xx:
  bet:=1:
  for i from 1 to NN do
  bet:=bet/beta[uint_of_x(xxt)];
  ud[i]:=a[uint_of_x(xxt)];
  udb[i]:=a[uint_of_x(xxt)]*bet;
  xxt:=uT(xxt);
  od:

  xxt:=xx:
  bet:=1:
  for i from 1 to NN do
  bet:=bet/beta[uint_of_x(xxt)];
  d[i]:=a[uint_of_x(xxt)];
  db[i]:=a[uint_of_x(xxt)]*bet;
  xxt:=T(xxt);
  od:
  print(ud);
  uls_it_x:=evalf(sum(udb[j1],j1=1..NN));
  print(d);
  ls_it_x:=evalf(sum(db[j1],j1=1..NN));
  terr:=xx-uls_it_x;
  err:=xx-ls_it_x;
```

$$Digits := 100$$

$$NN := 50$$

$$xx := 0.3957188605$$

[0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0, 0.5000000000, 0, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0.5000000000]

$uIs\_it\_x := 0.3957188605$

[0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0, 0.5000000000, 0, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0, 0, 0.5000000000, 0.5000000000, 0, 0.5000000000, 0.5000000000, 0.5000000000]

$Is\_it\_x := 0.3957188605$

$terr := 3.720957775\ 10^{-16}$

$err := 3.720957775\ 10^{-16}$ **(5)**

```
>
>
> NN:=50; chi:=(x1,x2,t)->piecewise(t<x1,0,t<=x2,1,0);
  uchi:=(x1,x2,t)->piecewise(t<x1,0,t<x2,1,0);

  #Expansion of c1, c2 ... and all the S's

  for i from 1 to Kc do
  xxt:=c[i];
  bet:=1:
       for n from 1 to NN+1 do
        if sidec[i]=1 then intx:=uint_of_x(xxt) else intx:=
  int_of_x(xxt) fi;
           bet_real:=bet;
           bet:=bet/beta[intx];
           dcb[i,n]:=a[intx]*bet;

         if sidec[i]=0 then
              for ii from 1 to Kc do
                   if xxt>c[ii] then cc[i,ii,n]:=1*bet_real else
  cc[i,ii,n]:=0 fi;
              od;
              if intx=1 then Sc[i,n]:= 0
                             else Sc[i,n]:=sum(1/(beta[j7]),j7=1..
  intx-1)*bet_real fi;
              #bc[i,n]:=b[intx]*bet_real;
              ################
                      else
```

```
                for ii from 1 to Kc do
                     if xxt<c[ii] then cc[i,ii,n]:=1*bet_real else
cc[i,ii,n]:=0 fi;
                od;
                 if intx=N then Sc[i,n]:= 0
                              else Sc[i,n]:=sum(1/(beta[j8]),j8=
intx+1..N)*bet_real fi;
                #bc[i,n]:=b[intx+1]*bet_real;
        fi;
        valc[i,n]:=xxt;
        betc[i,n]:=bet_real;
        if sidec[i]=1 then xxt:=uT(xxt) else xxt:=T(xxt)fi;
        od:
Is_it_x:=sum(dcb[i,j1],j1=1..NN);
od;
for i from 1 to Kc do
S[i]:=evalf(sum(Sc[i,j2+1],j2=1..NN));
#sum_b[i]:=evalf(sum(bc[i,j2+1],j2=1..NN));
od;
for i from 1 to Kc do
for j from 1 to Kc do
SS[i,j]:=evalf(sum(cc[i,j,j1+1],j1=1..NN));

#print(`SS[`,i,j,`] =`,SS[i,j]):
od; od:
#for n from 1 to 20 do

#print(1, 2, cc[1, 2, n]*beta[1], valc[1, n]);
#print(3, 2, cc[3, 2, n]*beta[2], valc[3, n]);
#od:
```

$$NN := 50$$

$$\chi := (x1, x2, t) \rightarrow piecewise(t < x1, 0, t \le x2, 1, 0)$$

$$uchi := (x1, x2, t) \rightarrow piecewise(t < x1, 0, t < x2, 1, 0)$$

$$xxt := 0.2500000000$$

$$bet := 1$$

$$Is\_it\_x := 0.2500000000$$

$$xxt := 0.7500000000$$

$$bet := 1$$

$$Is\_it\_x := 0.7500000000$$

$$S_1 := 0.5000000000$$

$$S_2 := 0.5000000000$$ **(6)**

```
>
>
```

```
MM:=matrix(Kc,Kc,[]):
for i from 1 to Kc do
for j from 1 to Kc do

MM[i,j]:=-SS[j,i];
od;od;
print(`MM = `,MM);



print(`eigenvalues MM =`,eigenvalues(MM));
print(`1/ average beta = `,1/ave_beta);
ve:=vector(Kc,[]):
for i from 1 to Kc do
ve[i]:=1;

MM[i,i]:=MM[i,i]+1.0;
od:

print(MM);
print(`det MM = `,det(MM));
print(ve);



DD:=linsolve(MM,ve);
sum((S[ii7]-1/beta[j_of_c[ii7]])*DD[ii7],ii7=1..Kc)-(1-sum
(1/beta[i8],i8=1..N));
ve0:=vector(Kc,[]):
for i from 1 to Kc do
ve[i]:=0;
od:
DD0:=linsolve(MM,ve0);
```

$$MM = , \begin{bmatrix} -1.0000000000 & -0.0000000000 \\ -0.0000000000 & -1.0000000000 \end{bmatrix}$$

$$eigenvalues\ MM =, -1.0000000000, -1.0000000000$$

$$1/\ average\ beta = , \frac{1}{ave\_beta}$$

$$\begin{bmatrix} 8.881784197\ 10^{-16} & -0.0000000000 \\ -0.0000000000 & 8.881784197\ 10^{-16} \end{bmatrix}$$

$$det\ MM = , 7.888609052\ 10^{-31}$$

$$\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$DD := \begin{bmatrix} 1.125899907\,10^{15} & 1.125899907\,10^{15} \end{bmatrix}$$

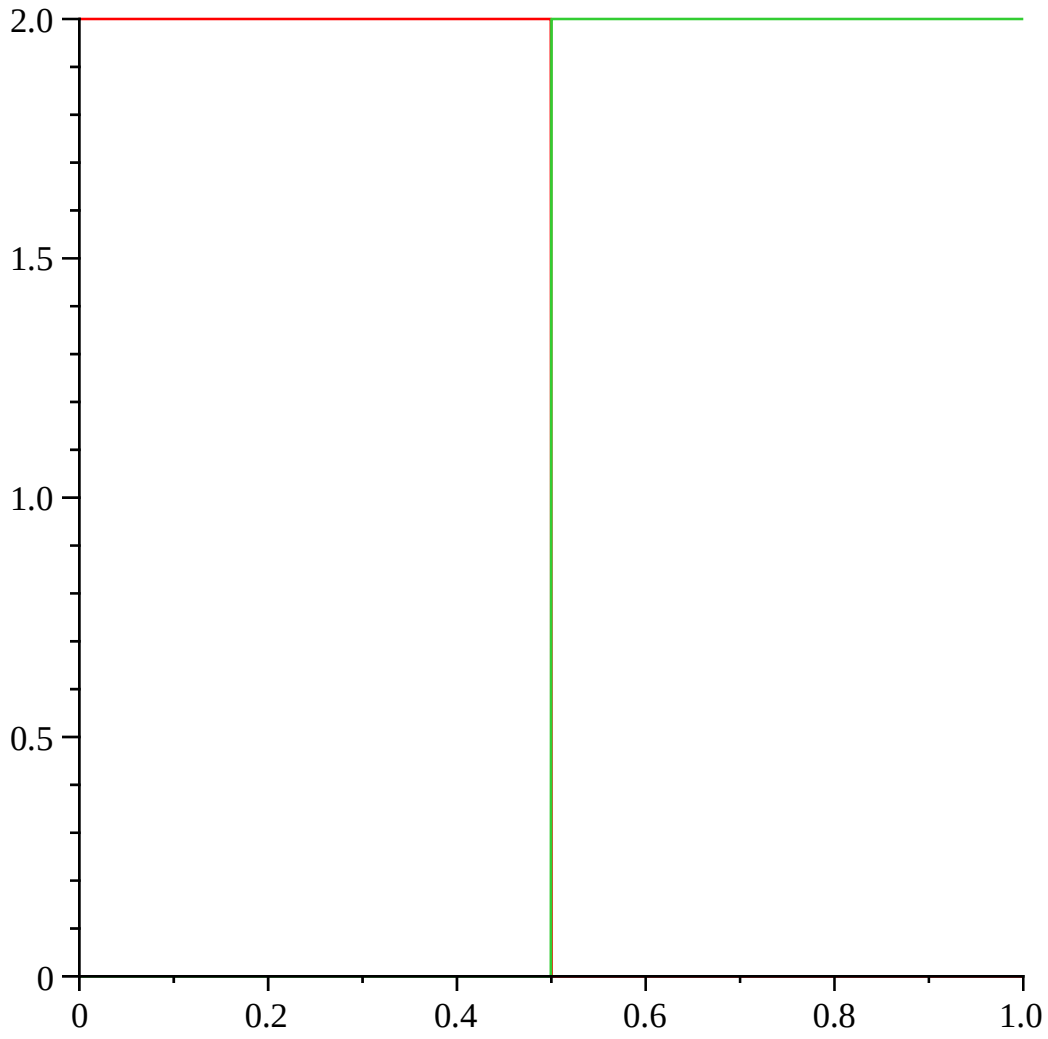
$$-0.5000000000$$

$$DD0 := \begin{bmatrix} 1.125899907\,10^{15}\,ve0_1 & 1.125899907\,10^{15}\,ve0_2 \end{bmatrix} \quad \textbf{(7)}$$

```
> denh1: =t - >bet a[ j _of _c[ 1] ] * sum( ( chi ( 0, val c[ 1, i 1+1] , t ) ) * bet c[ 1,
  i 1+1] , i 1=1. . 50) ;
  denh2: =t - >bet a[ j _of _c[ 2] ] * sum( ( chi ( val c[ 2, i 1+1] , 1, t ) ) * bet c[ 2,
  i 1+1] , i 1=1. . 50) ;

  pl ot ( [ denh1, denh2] , 0. . 1) ;
```

$$denh1 := t \rightarrow \beta_{j\_of\_c_1}\left(\sum_{i1=1}^{50} \chi\left(0, valc_{1,\,i1+1}, t\right)\, betc_{1,\,i1+1}\right)$$

$$denh2 := t \rightarrow \beta_{j\_of\_c_2}\left(\sum_{i1=1}^{50} \chi\left(valc_{2,\,i1+1}, 1, t\right)\, betc_{2,\,i1+1}\right)$$




```
>
> densi t y: =pr oc( t )  l ocal  j , i ,  den;
                    i : =' i ' :
```

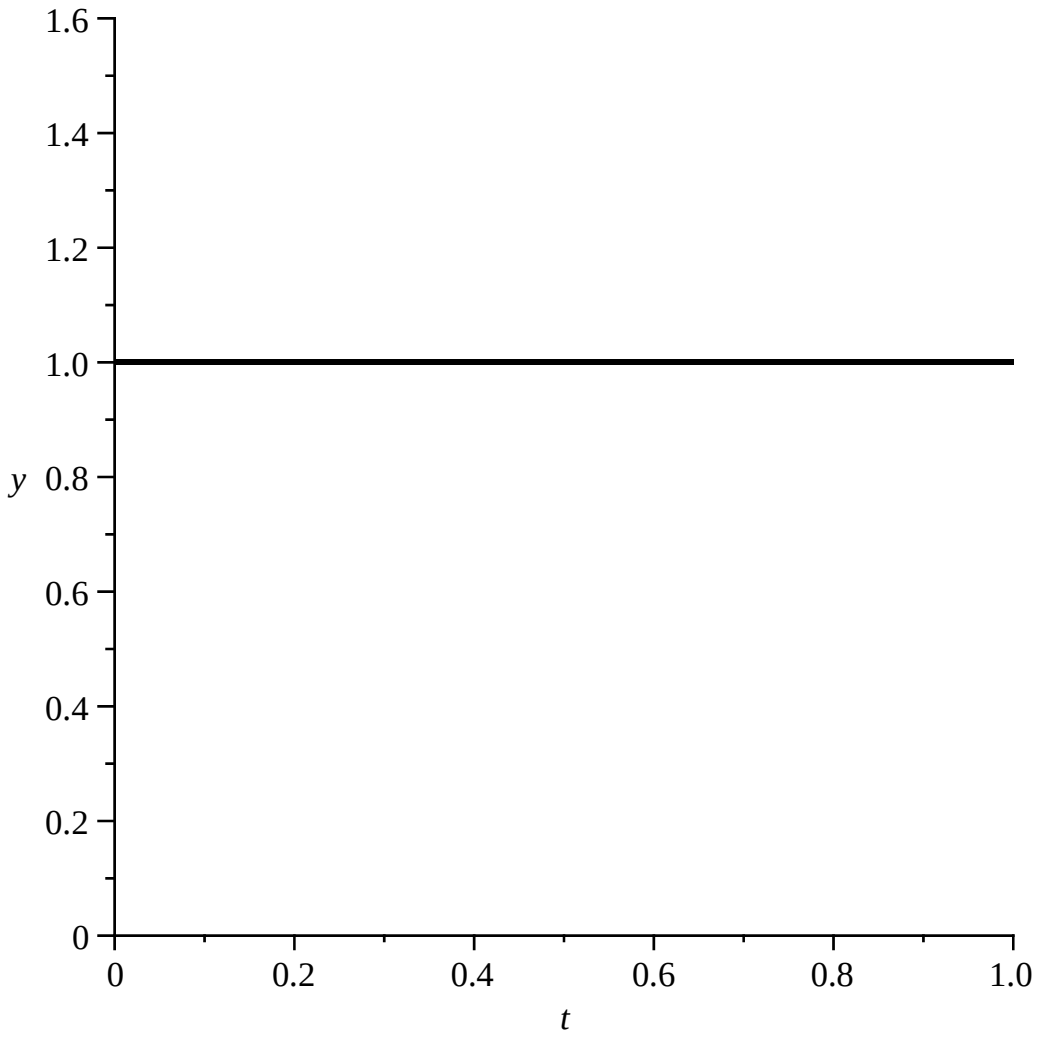
```
                    #den:=sum(chi(b[i],b[i+1],t)/beta[i],i=1..N);
                  den:=1:
                  for j from 1 to Kc do
                  if sidec[j]=0 then
                    den:=den+ DD[j]*sum((chi(0,valc[j,i1+1],t))*
betc[j,i1+1],i1=1..50) fi;

                    if sidec[j]=1 then
                     den:=den+ DD[j]*sum((uchi(valc[j,i1+1],1,t))*
betc[j,i1+1],i1=1..50) fi;


                    od;
                  return den;
            end proc;

#Normalizing factor

NC:=1:
for j from 1 to Kc do
if sidec[j]=0 then
 NC:=NC+DD[j]*sum((valc[j,i1+1])*betc[j,i1+1],i1=1..50) fi;
if sidec[j]=1 then
NC:=NC+DD[j]*sum((1-valc[j,i1+1])*betc[j,i1+1],i1=1..50) fi;

 od:

print(`NC = `,NC);

plot([(1/NC)*density(t),(1/NC)*density_t(t)],t=0..1-0.000001,y=
0..1.6,color=black,thickness=2);
```

```
density := proc(t)
    local j, i, den;
    i := 'i';
    den := 1;
    for j to Kc do
        if sidec[j] = 0 then
            den := den + DD[j] * (sum(chi(0, valc[j, i1 + 1], t) * betc[j, i1 + 1], i1 = 1 ..50))
        end if;
        if sidec[j] = 1 then
            den := den + DD[j] * (sum(uchi(valc[j, i1 + 1], 1, t) * betc[j, i1 + 1], i1 = 1 ..50))
        end if
    end do;
    return den
```

end proc

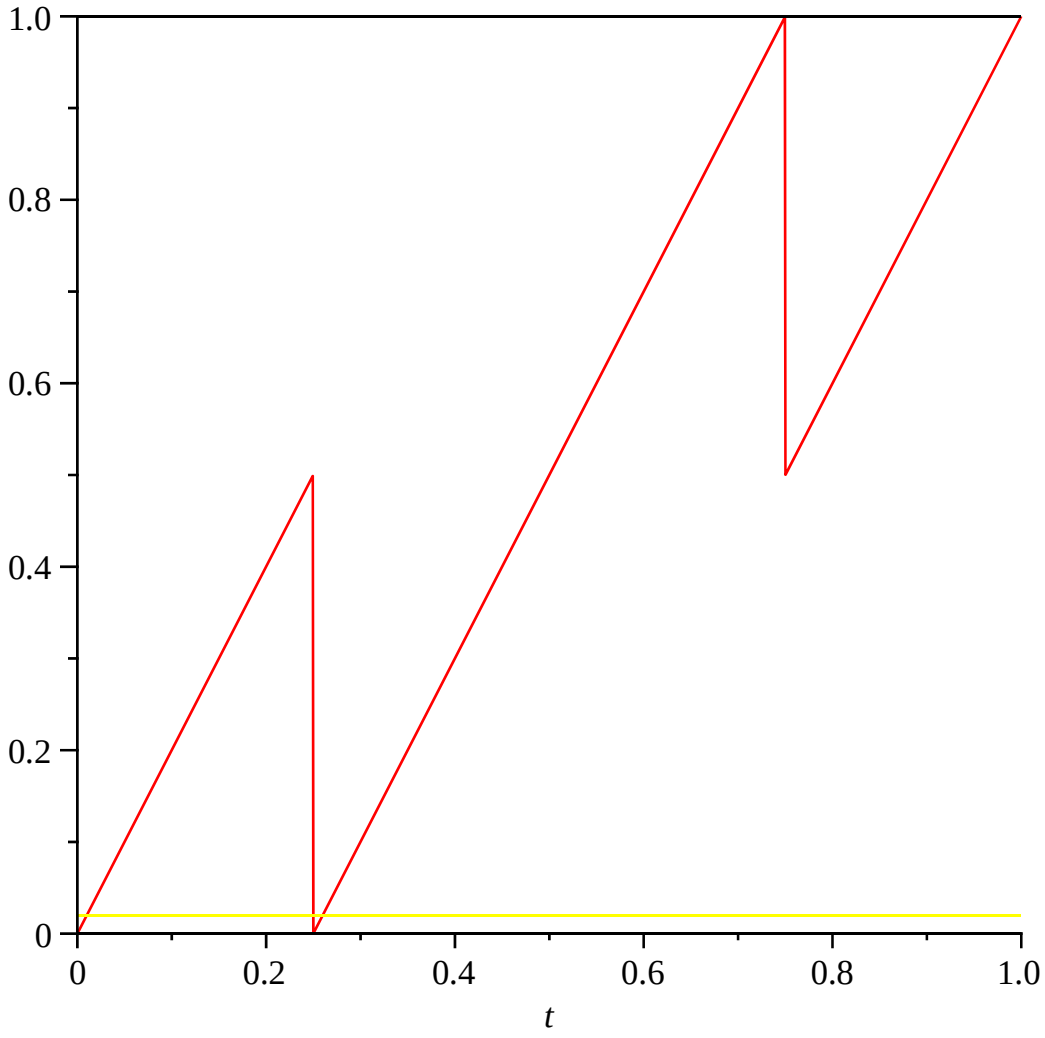
$$NC = , 1.125899907\ 10^{15}$$

```
Warning, unable to evaluate 1 of the 2 functions to numeric
values in the region; see the plotting command's help page to
ensure the calling sequence is correct
```




```
>
>
   #check density
   #preimages
   for j6 from 0 to 9 do
   y[j6]:=j6/10+(0.1)*rand()/10^12;
   od:
   for j6 from 0 to 9 do
   for i3 from 1 to N do
   pre[i3]:=(y[j6]+a[i3])/beta[i3];
   od;
   plot([T(t),0,1,y[j6]],t=0..1,
   color=[red,black,black,yellow]);
   su:=0:
   for i3 from 1 to N do
   if (pre[i3]>=b[i3] and pre[i3]<=b[i3+1]) then
```

```
  su: =su+eval f ( densi t y( pr e[ i 3] ) / bet a[ i 3] ) ;
pr i nt ( i 3) ;
f i ;
od;
er r [ j 6] : =eval f ( densi t y( y[ j 6] ) - su) ;
od;

f or  j 6  f r om 0  t o  9  do
pr i nt ( ` y  =` , y[ j 6] ) ;
pr i nt ( ` er r [ ` , j 6, ` ] =` , er r [ j 6] ) ;
od;
```

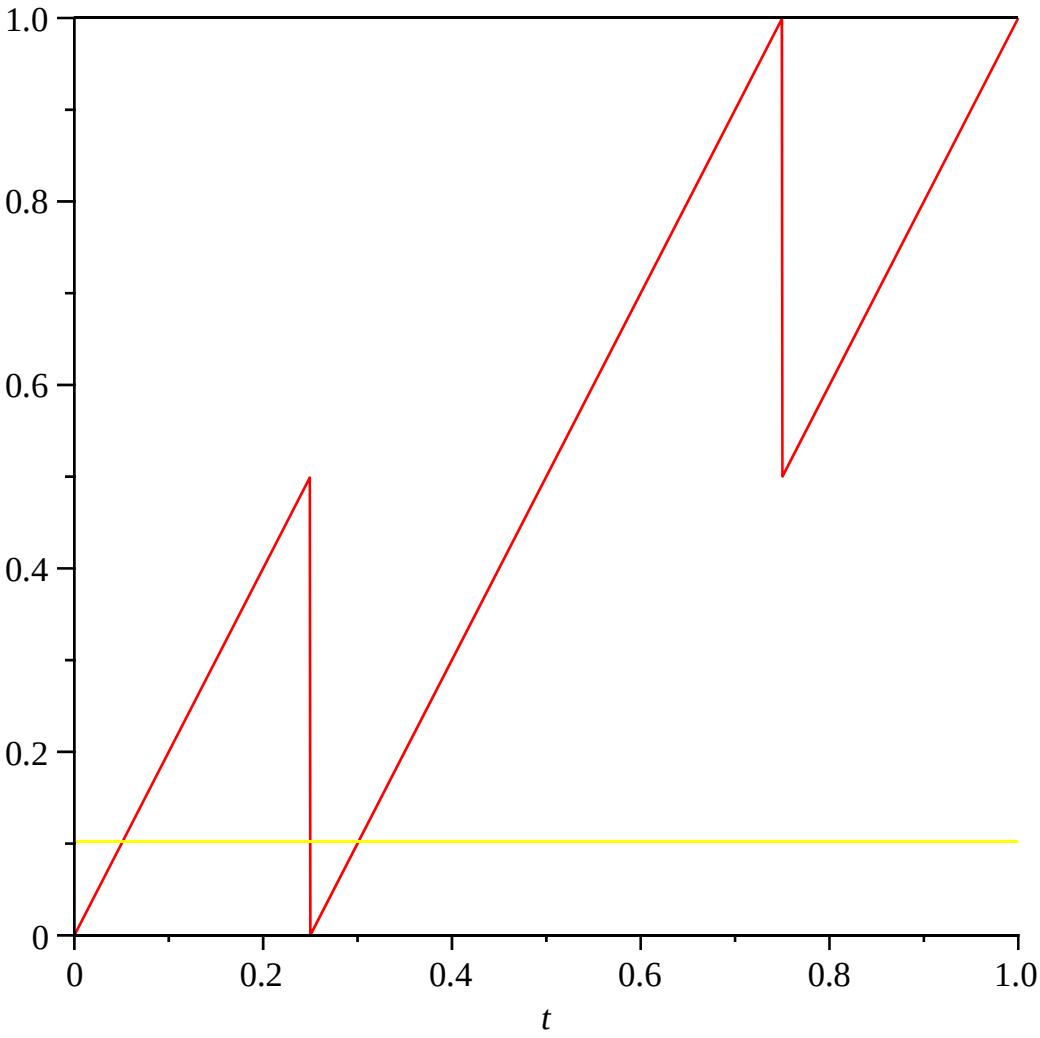


$su := 0$

1

2

$err_0 := 0.0000000000$

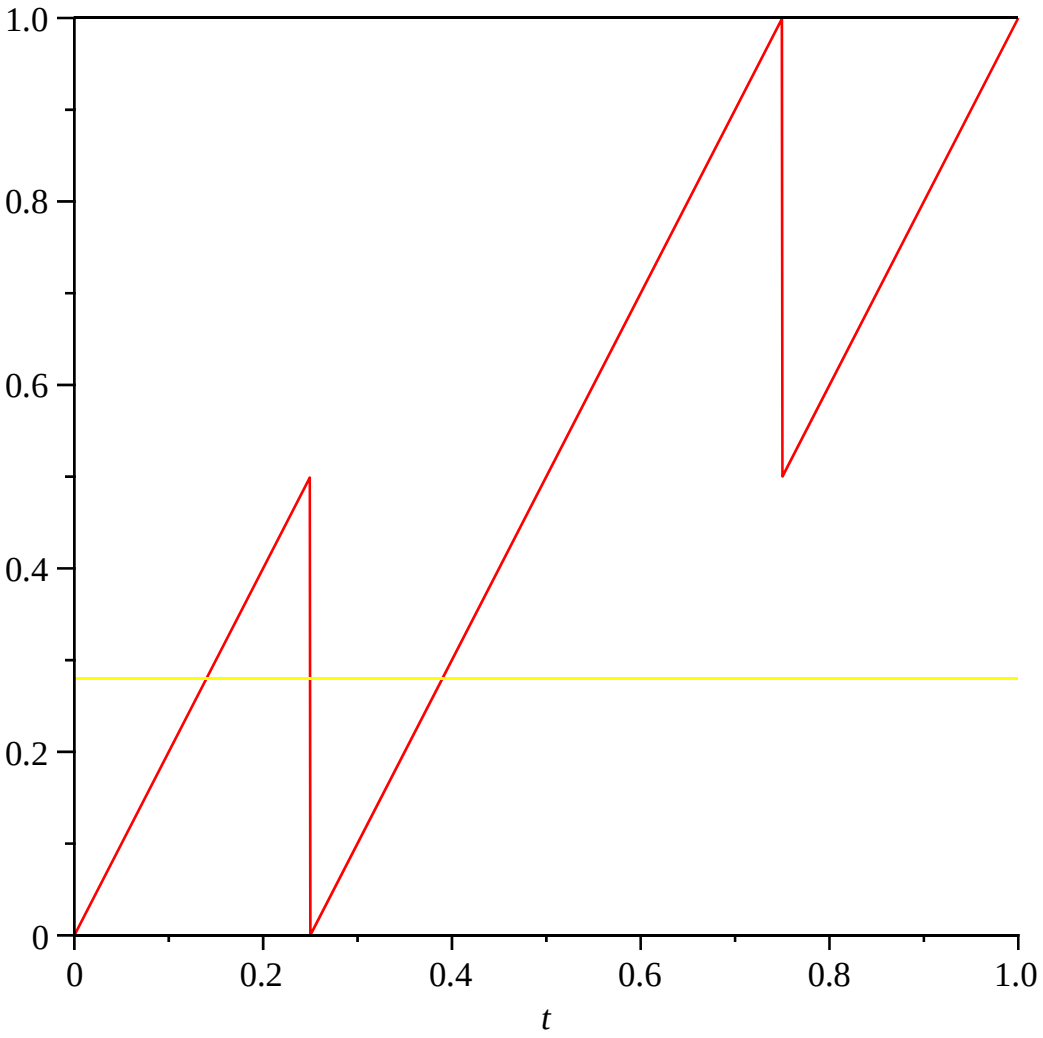


$su := 0$

1

2

$err_1 := 0.0000000000$

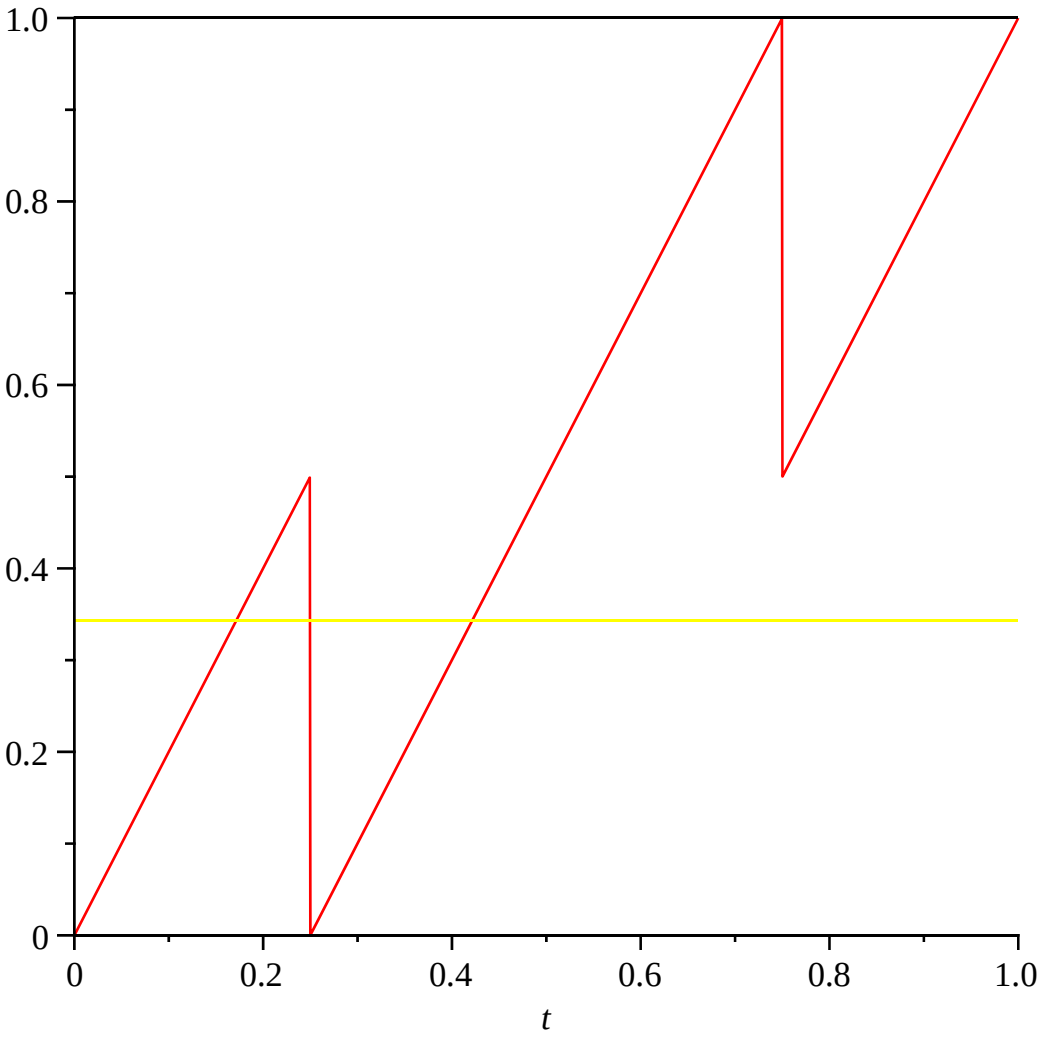


$su := 0$

1

2

$err_2 := 0.0000000000$

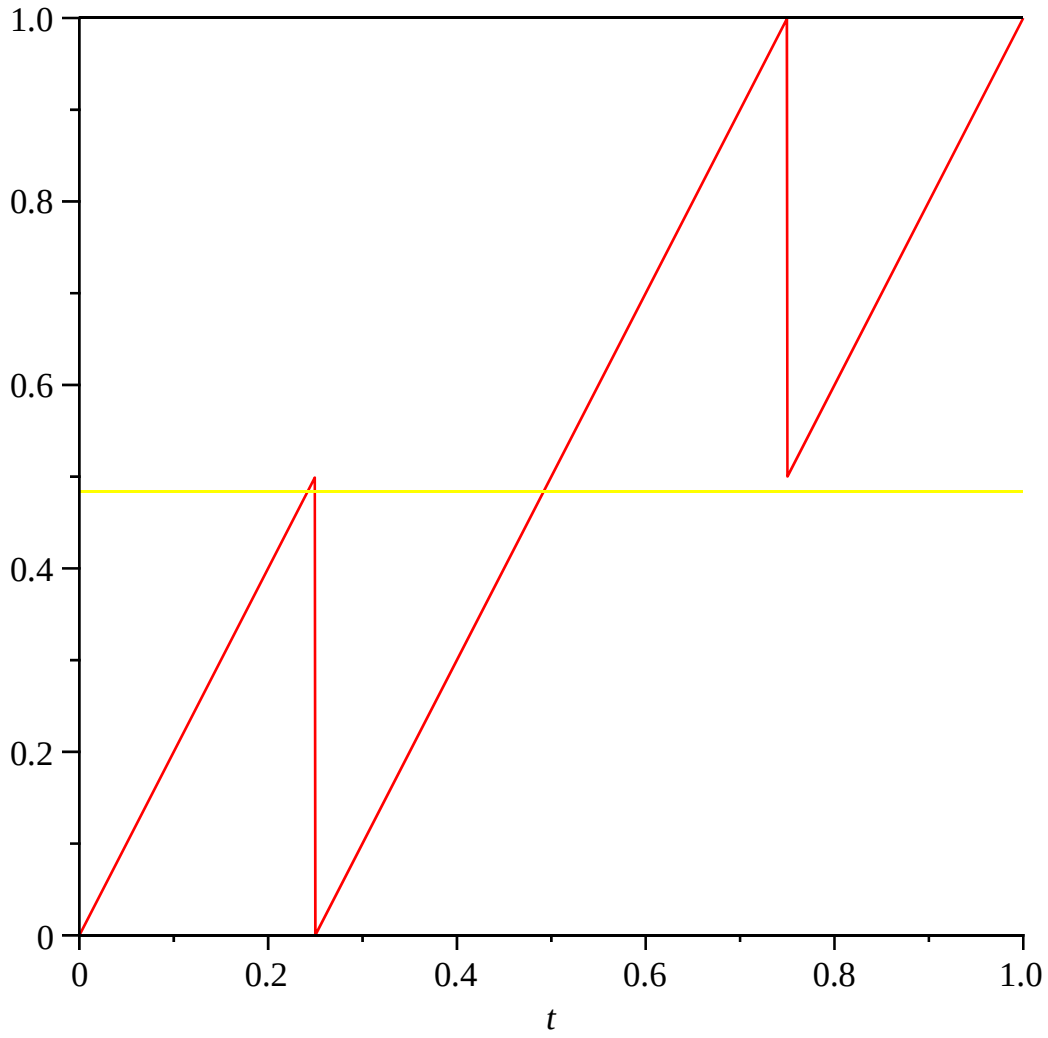


$su := 0$

1

2

$err_3 := 0.0000000000$

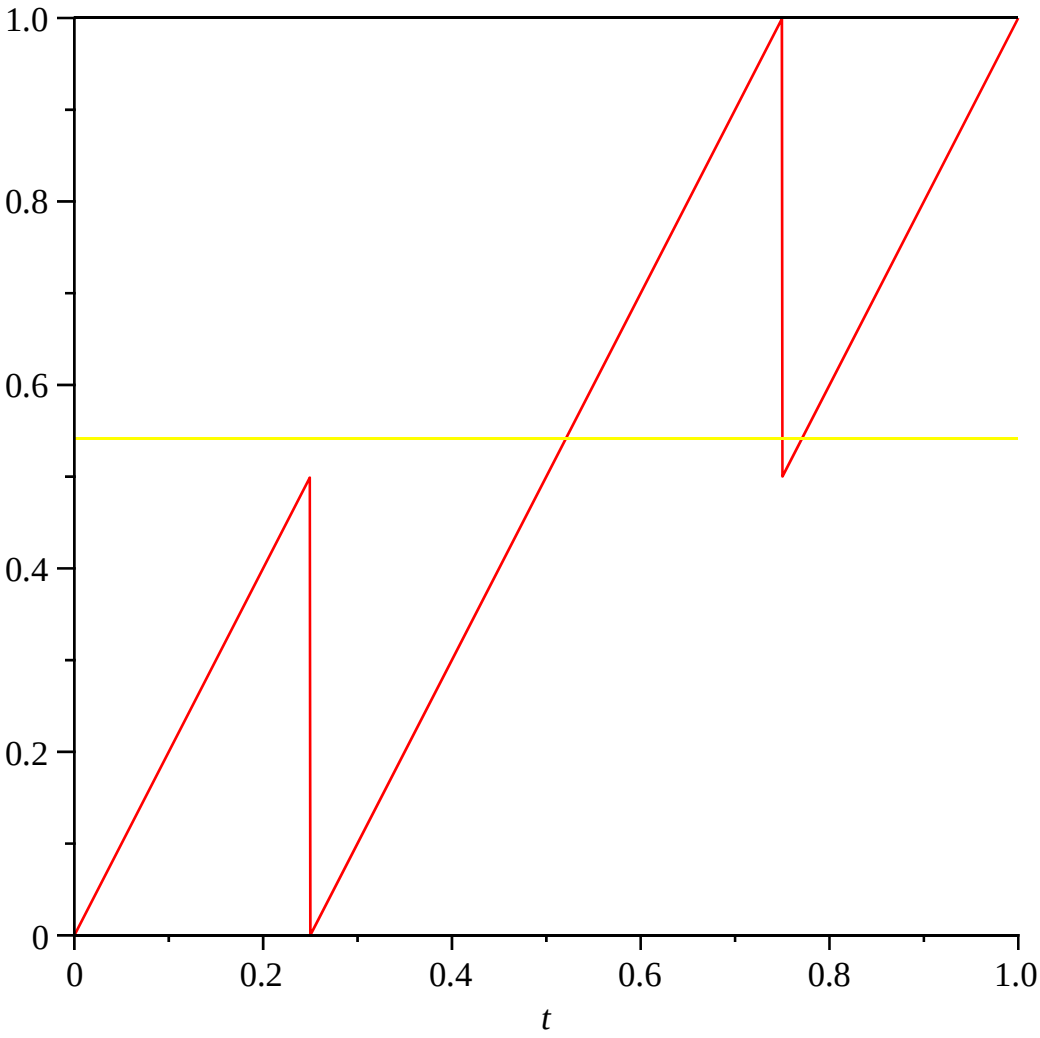


$su := 0$

1

2

$err_4 := 0.0000000000$

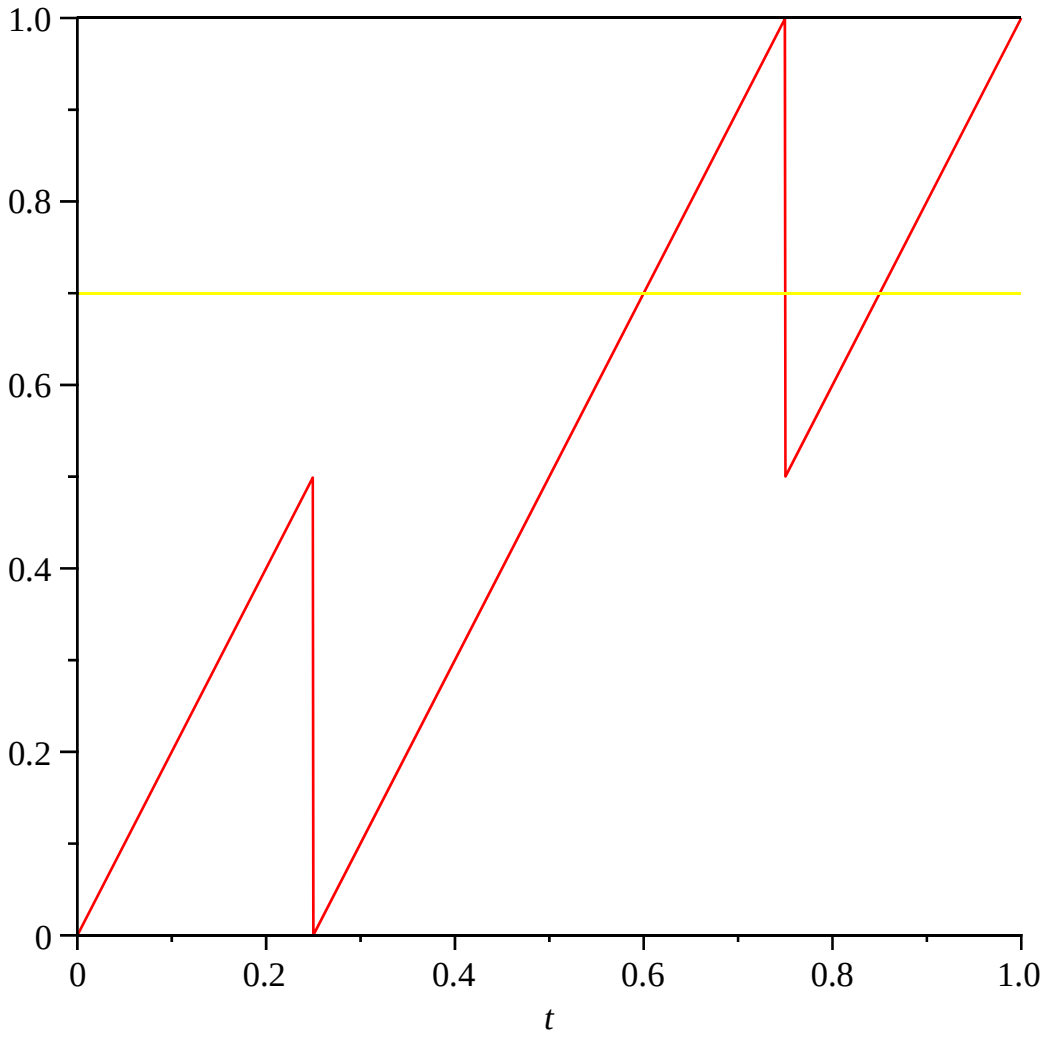


$su := 0$

2

3

$err_5 := 0.0000000000$

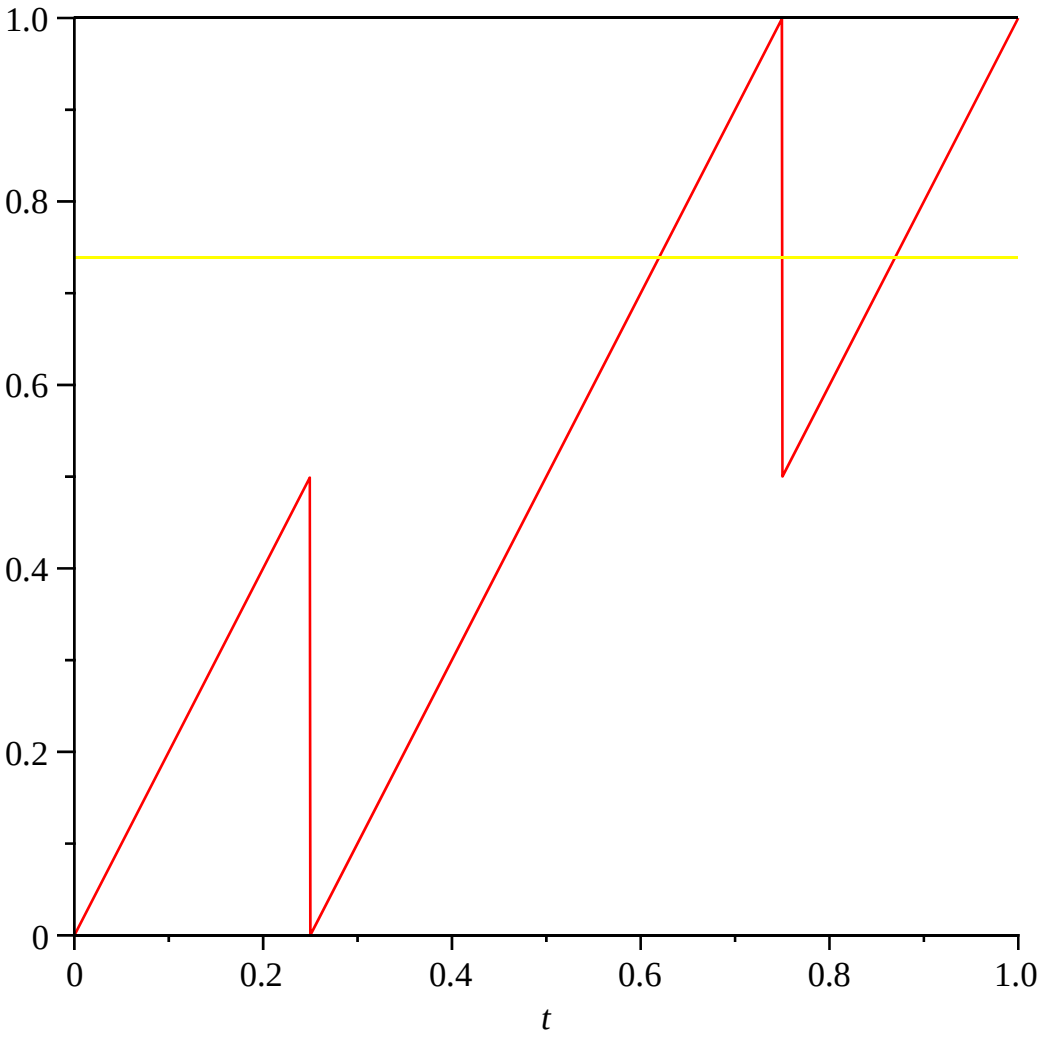


$su := 0$

2

3

$err_6 := 0.0000000000$

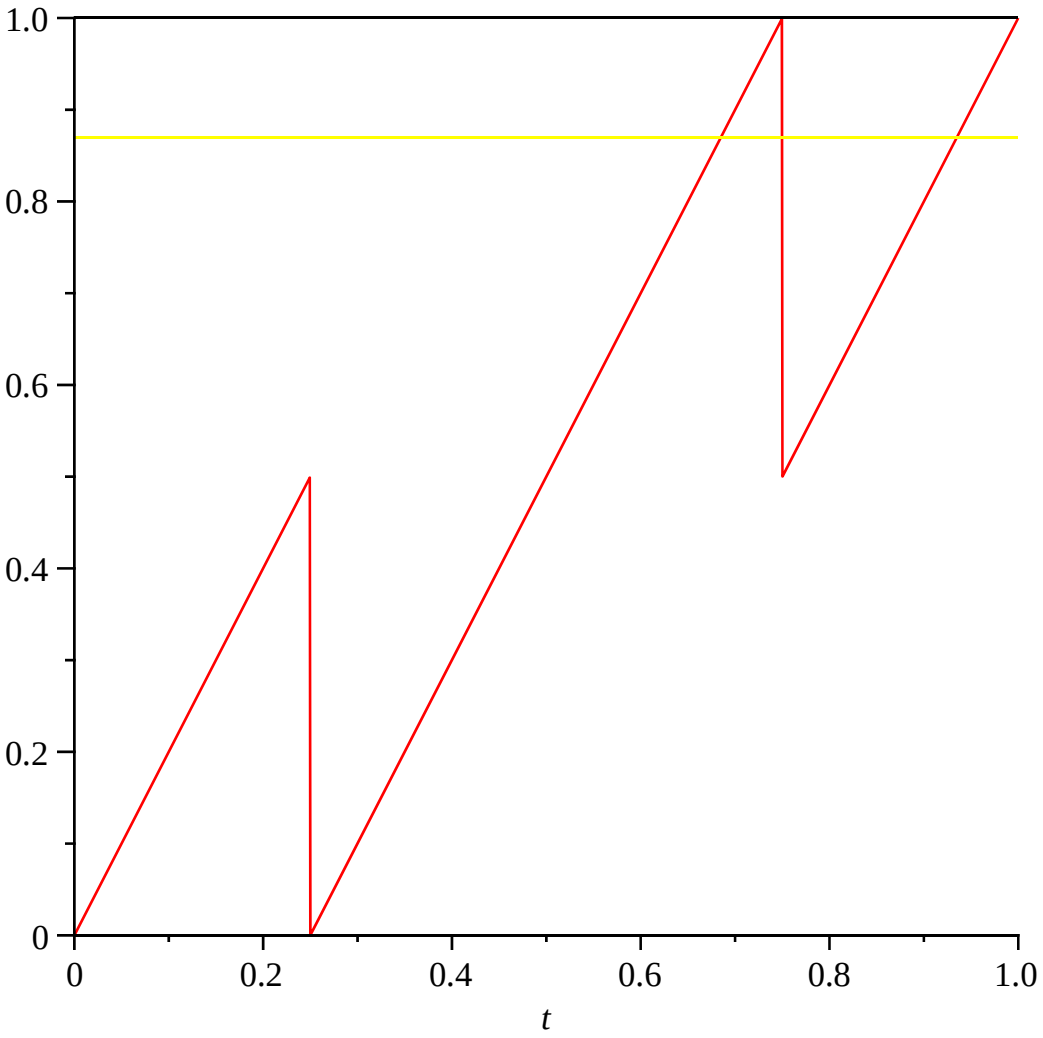


$su := 0$

2

3

$err_7 := 0.0000000000$

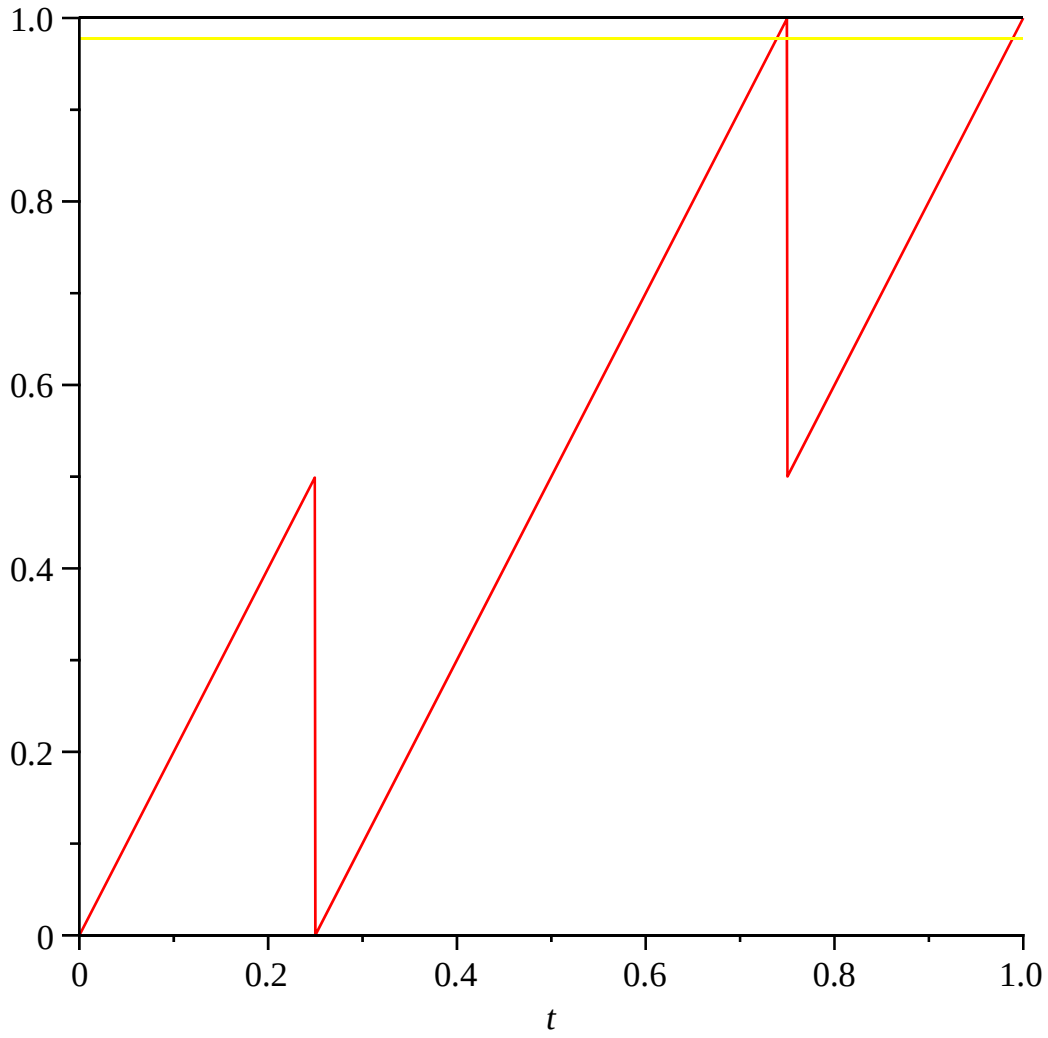


$su := 0$

2

3

$err_8 := 0.0000000000$



$su := 0$

2

3

$err_9 := 0.0000000000$

$y =, 0.0193139816$

$err[, 0,]=, 0.0000000000$

$y =, 0.1022424170$

$err[, 1,]=, 0.0000000000$

$y =, 0.2800187484$

$err[, 2,]=, 0.0000000000$

$y =, 0.3427552057$

$err[, 3,]=, 0.0000000000$

$y =, 0.4842622684$

$err[, 4,]=, 0.0000000000$

$y =, 0.5412286286$

$err[, 5,]=, 0.0000000000$

$y =, 0.6996417214$

$err[, 6,]=, 0.0000000000$

$y =, 0.7386408307$

err[, 7, *]*=, 0.0000000000

y =, 0.8694607189

err[, 8, *]*=, 0.0000000000

y =, 0.9773012980

err[, 9, *]*=, 0.0000000000

>