```
> with(plots):Digits:=100:interface(displayprecision=10):with
  (linalg):


> N:=9;
  KK:=9;
  ### Change of notation: Now the indices K[i] are in order not
  alpha's
  # vector U shows  if the branch is up (1) or down (0)
  K:=vector(N,[]):
  U:=vector(N,[]):
  UU:=vector(N,[]):
  alpha:=vector(N,[]):
  gamm=vector(N,[]):#heiths of lower ends of hanging branches
                          # alpha[i]+gamma[i]<1   !!!!!!!
                          #if gamma[i]>0 then U[i]:=0, UU[i]:=1
  for j from 1 to N do
  U[j]:=0.0:
  UU[j]:=0:
  gamm[j]:=0:
  od:
  alpha[1]:=0.3:K[1]:=1:U[K[1]]:=0.0:
  alpha[2]:=0.3:K[2]:=2:U[K[2]]:=0.0:#
  alpha[3]:=0.3:K[3]:=3:U[K[3]]:=0.0:#
  alpha[4]:=0.2:K[4]:=4:U[K[4]]:=0.0:gamm[K[4]]:=0.3:UU[K[4]]:=1:
  #
  alpha[5]:=0.2:K[5]:=5:U[K[5]]:=0.0:gamm[K[5]]:=0.3:UU[K[5]]:=1:
  alpha[6]:=0.2:K[6]:=6:U[K[6]]:=0.0:gamm[K[6]]:=0.3:UU[K[6]]:=1:
  alpha[7]:=0.5:K[7]:=7:U[K[7]]:=1.0:
  alpha[8]:=0.5:K[8]:=8:U[K[8]]:=1.0:
  alpha[9]:=0.5:K[9]:=9:U[K[9]]:=1.0:
  print(`alpha =`,alpha);
  print(`K =`,K);
  print(`U =`,U);
  print(`UU =`,UU);
  i:='i':
  beta:=N-KK+sum(alpha[i],i=1..KK);i:='i':
  delta1:=(xw,yw)-> piecewise(xw<=yw,0,1):
  bb:=vector(N+1,[]):
  for j from 1 to N do
  b[j]:=(j-1-sum((1-alpha[i])*delta1( j,K[i]) ,i=1..KK))/beta:
   od:  i:='i':
  b[N+1]:=1:
  ag:=vector(N,[]):
  al:=vector(N,[]):
  a:=vector(N,[]):
  c:=vector(N,[]):
```

```
for j from 1 to N do
bb[j]:=b[j];
ag[j]:=beta*b[j];
al[j]:=-1+beta*b[j+1];
od:
for j from 1 to N do
a[j]:=(j-1-sum((1-alpha[i])*delta1(j,K[i]-U[j]),i=1..KK)-gamm
[j]);
 od:

print(`b =`,bb);
print(`ag =`,ag);
print(`al =`,al);
print(`a =`,a);
print(`gamma =`,gamm);
```

$$N := 9$$

$$KK := 9$$

$alpha =, [0.3000000000, 0.3000000000, 0.3000000000, 0.2000000000, 0.2000000000, 0.2000000000, 0.5000000000, 0.5000000000, 0.5000000000]$

$$K =, \left[\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}\right]$$

$U =, [0.0000000000, 0.0000000000, 0.0000000000, 0.0000000000, 0.0000000000, 0.0000000000, 1.0000000000, 1.0000000000, 1.0000000000]$

$$UU =, \left[\begin{array}{ccccccccc} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{array}\right]$$

$$\beta := 3.0000000000$$

$b =, [0.0000000000, 0.1000000000, 0.2000000000, 0.3000000000, 0.3666666667, 0.4333333333, 0.5000000000, 0.6666666667, 0.8333333333, bb_{10}]$

$ag =, [0.0000000000, 0.3000000000, 0.6000000000, 0.9000000000, 1.1000000000, 1.3000000000, 1.5000000000, 2.0000000000, 2.5000000000]$

$al =, [-0.7000000000, -0.4000000000, -0.1000000000, 0.1000000000, 0.3000000000, 0.5000000000, 1.0000000000, 1.5000000000, 2.0000000000]$

$a =, [0.0000000000, 0.3000000000, 0.6000000000, 0.6000000000, 0.8000000000, 1.0000000000, 1.0000000000, 1.5000000000, 2.0000000000]$

$$gamma =, \left[\begin{array}{ccccccccc} 0 & 0 & 0 & 0.3000000000 & 0.3000000000 & 0.3000000000 & 0 & 0 & 0 \end{array}\right]$$

```
> # ag shows maximal digit (greedy)
  #  al shows minimal digit (lazy)  #### if ag[j]=al[j] then  j is
  onto branch and there is
                                                                  #
  no choice there
  # a shows digits assigned automatically using the vector U: U(j)
  =1 lazy
  #                                                          U(j)=
```

```
   0 greedy
   # we can assign digit a arbitrarily between minimum and maximum
   and then put 2 into vector U

   #a[3]:=1.0: U[3]:=2.0:
   #a[5]:=1.90:U[5]:=2.0:
   #a[6]:=2.70:U[6]:=2.0:
   #print(`U =`,U);
   #print(`a =`,a);

   # Now we will name points c[i] (there is KK + number of 2's in U
   points c[i])
   # and create a vectors sidec[], ineqc[],signc[] which shows the
   character of the point c[i]
   Kc:=KK:# new number of c points
   for j from 1 to N do if UU[j]=1 then Kc:=Kc+1 fi od:
   print(`Kc =`,Kc);
   c:=vector(2*N,[]):
   sidec:=vector(2*N,[]):# 1 left (use uT), 0 right (use T)


   cj:=1:# this is the new index for c points
   for j from 1 to KK do
   if (U[K[j]]=0 and UU[K[j]]=0) then   c[cj]:=b[K[j]+1]; sidec[cj]
   :=0;cj:=cj+1 fi;
    if U[K[j]]=1 then  c[cj]:=b[K[j]]; sidec[cj]:=1;cj:=cj+1 fi;
    if (U[K[j]]=0 and UU[K[j]]=1) then  c[cj]:=b[K[j]]; sidec[cj]:=
   1;cj:=cj+1 ;
                        c[cj]:=b[K[j]+1]; sidec[cj]:=0;cj:=cj+1 ;
   fi;
   od;
   print(`c =`,c);
   print(`sidec =`,sidec);


>
   maa:=a[2]-a[1]:# maximum a[i+1]-a[i]
   for i from 3 to N do
   if (a[i]-a[i-1])>maa then maa:=(a[i]-a[i-1]) fi
   od;#
   maa;
   beta_max:=evalf(1+(a[N]-a[1])/maa);
> if beta> beta_max then print("ERROR") fi;


>
   uint_of_x:=x->piecewise(x<b[2],1,# This function needs additions
```

```
by hand for
                                        # N>9 . Automathic procedure
causes plotting problems
                                        # but is used in other
programs
                            x<b[3], 2,
                            x<b[4], 3,
                            x<b[5], 4,
                            x<b[6], 5,
                            x<b[7], 6,
                            x<b[8], 7,
                            x<b[9], 8,
                                  9);
int_of_x:=x->piecewise(x<=b[2], 1,# This function needs additions
by hand for
                                        # N>9 . Automathic procedure
causes plotting problems
                                        # but is used in other
programs
                            x<=b[3], 2,
                            x<=b[4], 3,
                            x<=b[5], 4,
                            x<=b[6], 5,
                            x<=b[7], 6,
                            x<=b[8], 7,
                            x<=b[9], 8,
                                  9);
x:='x':
uT:=x->beta*x-a[uint_of_x(x)];
T:=x->beta*x-a[int_of_x(x)];
Tc:=vector(2*N,[]):
for j from 1 to Kc do
if sidec[j]=0 then    Tc[j]:=T(c[j]);
              else    Tc[j]:=uT(c[j])fi;
od:
print(`Tc = `, Tc);

plot([uT(x), x, 0, 1, 1-alpha[1], 1-alpha[2]], x=0..1, thickness=[2, 1,
1, 1, 1, 1, 1]);
plot([T(x), x, 0, 1, alpha[1], alpha[2], 0.2], x=0..1, thickness=[2, 1, 1,
1, 1, 1, 1, 1]);
```

$$Kc =, 12$$

$$c =, [0.1000000000, 0.2000000000, 0.3000000000, 0.3000000000, 0.3666666667,$$

$$0.3666666667, 0.4333333333, 0.4333333333, 0.5000000000, 0.5000000000,$$

$0.6666666667, 0.8333333333, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}\Big]$

$sidec =,$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & sidec_{13} & sidec_{14} & sidec_{15} & sidec_{16} & sidec_{17} & sidec_{18} \end{bmatrix}$$

$$0.5000000000$$

$$\beta max := 5.0000000000$$

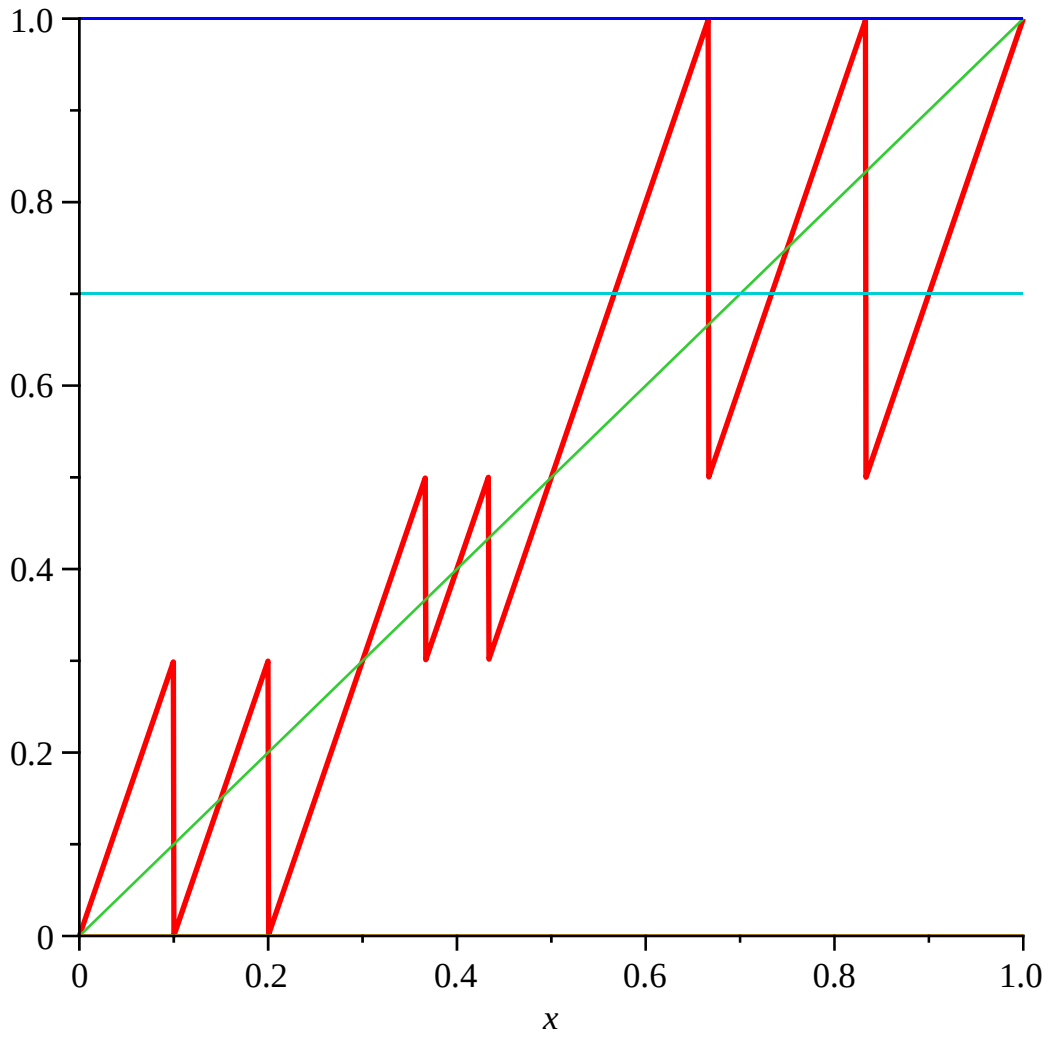
$uint\_of\_x := x \rightarrow piecewise\left(x < b_2, 1, x < b_3, 2, x < b_4, 3, x < b_5, 4, x < b_6, 5, x < b_7, 6, x < b_8, 7, x < b_9, 8, 9\right)$

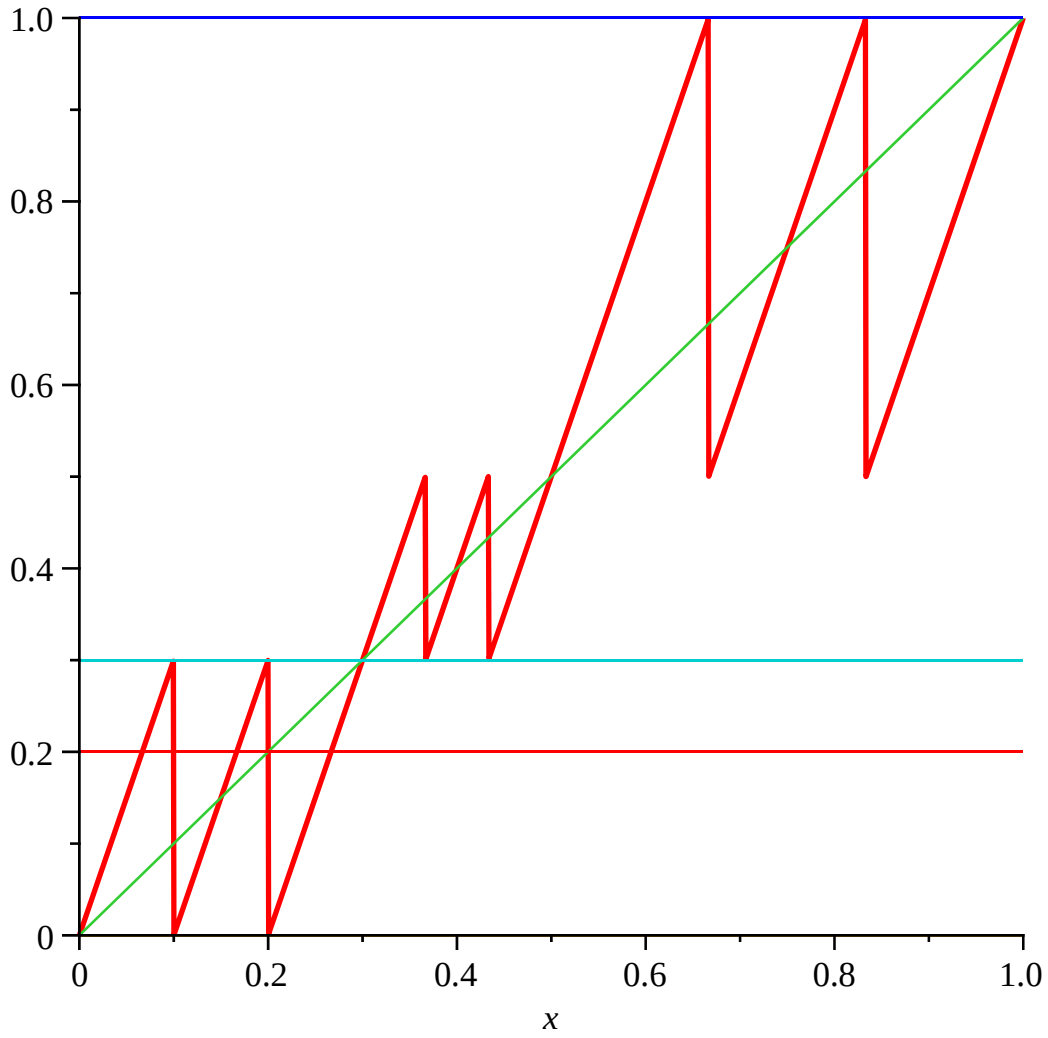
$int\_of\_x := x \rightarrow piecewise\left(x \le b_2, 1, x \le b_3, 2, x \le b_4, 3, x \le b_5, 4, x \le b_6, 5, x \le b_7, 6, x \le b_8, 7, x \le b_9, 8, 9\right)$

$$uT := x \rightarrow \beta\, x - a_{uint\_of\_x(x)}$$

$$T := x \rightarrow \beta\, x - a_{int\_of\_x(x)}$$

$Tc = , \Big[0.3000000000, 0.3000000000, 0.3000000000, 0.3000000000, 0.5000000000, 0.3000000000, 0.5000000000, 0.3000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, Tc_{13}, Tc_{14}, Tc_{15}, Tc_{16}, Tc_{17}, Tc_{18}\Big]$






```
>
> ud:=vector(50):Digits:=100;NN:=50;
  d:=vector(50):
  xx:=c[3];#evalf(rand()/10^12);
  xxt:=xx:
  for i from 1 to NN do
  ud[i]:=a[uint_of_x(xxt)];
  xxt:=uT(xxt);
  od:
  xxt:=xx:
  for i from 1 to NN do
  d[i]:=a[int_of_x(xxt)];
  xxt:=T(xxt);
  od:
  print(ud);
  uls_it_x:=evalf(sum(ud[j1]/beta^j1,j1=1..NN));
  print(d);
  ls_it_x:=evalf(sum(d[j1]/beta^j1,j1=1..NN));
  terr:=xx-uls_it_x;
  err:=xx-ls_it_x;
```

$$Digits := 100$$

$$NN := 50$$

$$xx := 0.3000000000$$

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]

$$uIs\_it\_x := 0.3000000000$$

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]

$$Is\_it\_x := 0.3000000000$$

$$terr := 4.178866707\ 10^{-25}$$

$$err := 4.178866707\ 10^{-25} \quad \textbf{(1)}$$

```
>
> NN:=50; chi:=(x1,x2,t)->piecewise(t<x1,0,t<=x2,1,0);
  uchi:=(x1,x2,t)->piecewise(t<x1,0,t<x2,1,0);

  #Expansion of c1, c2 ... and all the S's

  for i from 1 to Kc do
  xxt:=c[i];
       for n from 1 to NN+1 do
        if sidec[i]=1 then intx:=uint_of_x(xxt) else intx:=
  int_of_x(xxt) fi;
          dc[i,n]:=a[intx];
          ic[i,n]:=intx-1;
         if sidec[i]=0 then
              for ii from 1 to Kc do
                   if xxt>c[ii] then cc[i,ii,n]:=1 else cc[i,ii,n]
  :=0 fi;
```

```
                od;
                        else
                for ii from 1 to Kc do
                     if xxt<c[ii] then cc[i,ii,n]:=1 else cc[i,ii,n]
:=0 fi;
                od;
        fi;
        valc[i,n]:=xxt;
        if sidec[i]=1 then xxt:=uT(xxt) else xxt:=T(xxt)fi;
        od:
Is_it_x:=sum(dc[i,j1]/beta^j1,j1=1..NN);
S[i]:=sum(ic[i,j1+1]/beta^(j1+1),j1=1..NN);
od;
for i from 1 to Kc do
for j from 1 to Kc do
SS[i,j]:=sum(cc[i,j,j1+1]/beta^(j1+1),j1=1..NN);

#print(`SS[`,i,j,`] =`,SS[i,j]):
od;od:
```

$$NN := 50$$

$$\chi := (x1, x2, t) \rightarrow piecewise(t < x1, 0, t \leq x2, 1, 0)$$

$$uchi := (x1, x2, t) \rightarrow piecewise(t < x1, 0, t < x2, 1, 0)$$

$$xxt := 0.1000000000$$

$$Is\_it\_x := 0.1000000000$$

$$S_1 := 0.3333333333$$

$$xxt := 0.2000000000$$

$$Is\_it\_x := 0.2000000000$$

$$S_2 := 0.3333333333$$

$$xxt := 0.3000000000$$

$$Is\_it\_x := 0.3000000000$$

$$S_3 := 0.3333333333$$

$$xxt := 0.3000000000$$

$$Is\_it\_x := 0.3000000000$$

$$S_4 := 0.5000000000$$

$$xxt := 0.3666666667$$

$$Is\_it\_x := 0.3666666667$$

$$S_5 := 0.8333333333$$

$$xxt := 0.3666666667$$

$$Is\_it\_x := 0.3666666667$$

$$S_6 := 0.5000000000$$

$$xxt := 0.4333333333$$

$$Is\_it\_x := 0.4333333333$$

$$S_7 := 0.8333333333$$

$$xxt := 0.4333333333$$

$$Is\_it\_x := 0.4333333333$$

$$S_8 := 0.5000000000$$

$$xxt := 0.5000000000$$

$$Is\_it\_x := 0.5000000000$$

$$S_9 := 0.8333333333$$

$$xxt := 0.5000000000$$

$$Is\_it\_x := 0.5000000000$$

$$S_{10} := 1.0000000000$$

$$xxt := 0.6666666667$$

$$Is\_it\_x := 0.6666666667$$

$$S_{11} := 1.0000000000$$

$$xxt := 0.8333333333$$

$$Is\_it\_x := 0.8333333333$$

$$S_{12} := 1.0000000000$$

```
>
MM:=matrix(Kc,Kc,[]):
for i from 1 to Kc do
for j from 1 to Kc do

MM[j,i]:=-SS[i,j];
od;od;
print(`MM = `,MM);
print(`1/beta =`, 1/beta);

print(`eigenvalues MM =`,eigenvalues(MM));

ve:=vector(Kc,[]):
for i from 1 to Kc do
ve[i]:=1/beta;

MM[i,i]:=MM[i,i]+1/beta;
od:

print(MM);
print(ve);



DD:=linsolve(MM,ve);
```

MM = , [[−0.1666666667, −0.1666666667, −0.1666666667, −0.0000000000,
−0.1666666667, −0.0000000000, −0.1666666667, −0.0000000000, −0.1666666667,
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−0.1666666667, −0.0000000000, −0.1666666667, −0.0000000000, −0.1666666667,
−0.1666666667, −0.1666666667]]

1/beta =, 0.3333333333

eigenvalues MM =, 0.0000000000, −0.3333333333, 0.0000000000, −0.8333333333,

0.1666666667, $3.482478462\ 10^{-18}$, −0.3333333333, 0.0000000000, $1.040834086\ 10^{-17}$,
0.0000000000, 0.0000000000, 0.0000000000

[[0.1666666667, −0.1666666667, −0.1666666667, −0.0000000000, −0.1666666667,
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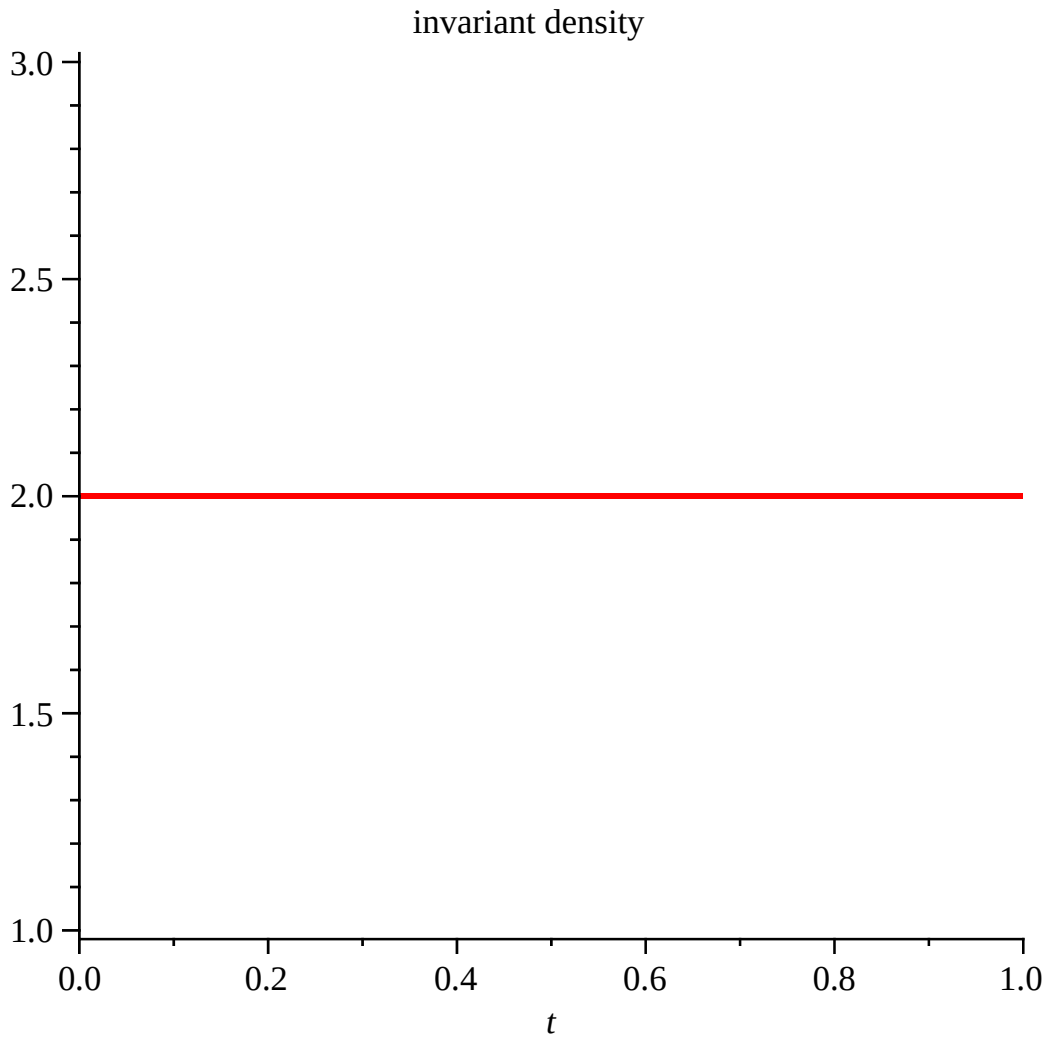
−0.1666666667, −0.0000000000, −0.1666666667, −0.0000000000, −0.1666666667,
−0.1666666667, 0.1666666667]]

[0.3333333333, 0.3333333333, 0.3333333333, 0.3333333333, 0.3333333333, 0.3333333333,
0.3333333333, 0.3333333333, 0.3333333333, 0.3333333333, 0.3333333333,

```
    0.3333333333]
DD := [-1.0000000000, -1.0000000000, -9.286370461 10^-25, -9.286370461 10^-25,                (2)
    -1.0000000000, -1.0000000000, -1.0000000000, -1.0000000000, -9.286370461 10^-25,
    -9.286370461 10^-25, -1.0000000000, -1.0000000000]
>
        density:=proc(t) local j, den;
                    den:=1/beta;
                  for j from 1 to Kc do
                  if sidec[j]=0 then
                    den:=den+ DD[j]*sum((chi(0,valc[j,i1+1],t))
  /beta^(i1+1),i1=1..50) fi;

                    if sidec[j]=1 then
                    den:=den+ DD[j]*sum((uchi(valc[j,i1+1],1,t))
  /beta^(i1+1),i1=1..50) fi;


                    od;
                  return den;
            end proc;
  #Normalizing factor

  NC:=1/beta:
  for j from 1 to KK do
  if sidec[j]=0 then
   NC:=NC+DD[j]*sum((valc[j,i1+1])/beta^(i1+1),i1=1..50) fi;
  if sidec[j]=1 then
  NC:=NC+DD[j]*sum((1-valc[j,i1+1])/beta^(i1+1),i1=1..50) fi;

   od:

  print(`NC = `,NC);

  plot([(1/NC)*density(t)],t=0..1-0.00000000001,title="invariant
  density",thickness=2);
density := proc(t)
  local j, den;
  den := 1/beta;
  for j to Kc do
    if sidec[j] = 0 then
      den := den + DD[j]*(sum(chi(0, valc[j, i1 + 1], t)/beta^(i1 + 1), i1 = 1..50))
    end if;
    if sidec[j] = 1 then
      den := den + DD[j]*(sum(uchi(valc[j, i1 + 1], 1, t)/beta^(i1 + 1), i1 = 1..50))
```

```
        end if
    end do;
    return den
end proc
```

$$NC = , -0.1666666667$$




```
>
>
#check density greedy
#preimages
for j6 from 1 to KK-1 do
y[j6]:=alpha[j6]+(alpha[j6+1]-alpha[j6])*rand()/10^12;
od:
y[0]:=alpha[1]*rand()/10^12:
y[KK]:=alpha[KK]+(1-alpha[KK])*rand()/10^12;
for j6 from 0 to KK do
for i3 from 1 to N do
pre[i3]:=(y[j6]+a[i3])/beta;
od;
#plot([T(t),0,1,y[j6],tT(tc[1]),tT(tc[2])],t=0..1,color=[red,
black,black,green,yellow,yellow]);
su:=0:
```

```
for i3 from 1 to N do
if (pre[i3]>=b[i3] and pre[i3]<=b[i3+1]) then
 su:=su+density(pre[i3])/beta;
print(i3);
fi;
od;
err[j6]:=density(y[j6])-su;
od;

for j6 from 0 to KK do
print(`y =`,y[j6]);
print(`err[`,j6,`]=`,err[j6]);
od;
```

$$y_9 := 0.8473035946$$

$$su := 0$$

1

2

3

$$err_0 := -1.\ 10^{-100}$$

$$su := 0$$

1

2

3

4

5

6

$$err_1 := 0.2222222222$$

$$su := 0$$

1

2

3

4

5

6

$$err_2 := 0.2222222222$$

$$su := 0$$

1

2

3

$$err_3 := -1.\ 10^{-100}$$

$$su := 0$$

1

2

3

$err_4 := -1.\ 10^{-100}$

$su := 0$

1

2

3

$err_5 := -1.\ 10^{-100}$

$su := 0$

4

5

6

$err_6 := 0.0000000000$

$su := 0$

4

5

6

7

8

9

$err_7 := 0.2222222222$

$su := 0$

4

5

6

7

8

9

$err_8 := 0.2222222222$

$su := 0$

7

8

9

$err_9 := 1.\ 10^{-100}$

$y =, 0.1159224922$

$err[, 0,]=, -1.\ 10^{-100}$

$y =, 0.3000000000$

$err[, 1,]=, 0.2222222222$

$y =, 0.3000000000$

$err[, 2,]=, 0.2222222222$

$y =, 0.2977575830$

$err[, 3,]=, -1.\ 10^{-100}$

$y =, 0.2000000000$

$err[, 4,]=, -1.\ 10^{-100}$

$y =, 0.2000000000$

$err[, 5,]=, -1.\ 10^{-100}$

$y =, 0.4527868053$

$err[, 6,]=, 0.0000000000$

$y =, 0.5000000000$

$err[, 7,]=, 0.2222222222$

$y =, 0.5000000000$

$err[, 8,]=, 0.2222222222$

$y =, 0.8473035946$

$err[, 9,]=, 1.\ 10^{-100}$

```
> #check density greedy
#preimages
y:=evalf(rand()/10^12);#0.50100400000000;
for i2 from 1 to N do
pre[i2]:=(y+a[i2])/beta;
od;
plot([T(t),0,1,y,T(c[1]),T(c[2])],t=0..1,color=[red,black,black,
green,yellow,yellow]);
su:=0:
for i2 from 1 to N do
if (pre[i2]>=b[i2] and pre[i2]<=b[i2+1]) then
 su:=su+density(pre[i2])/beta;
print(i2);
fi;
od;
err2:=density(y)-su;
```

$y := 0.7730129800$

$pre_1 := 0.2576709933$

$pre_2 := 0.3576709933$

$pre_3 := 0.4576709933$

$pre_4 := 0.4576709933$

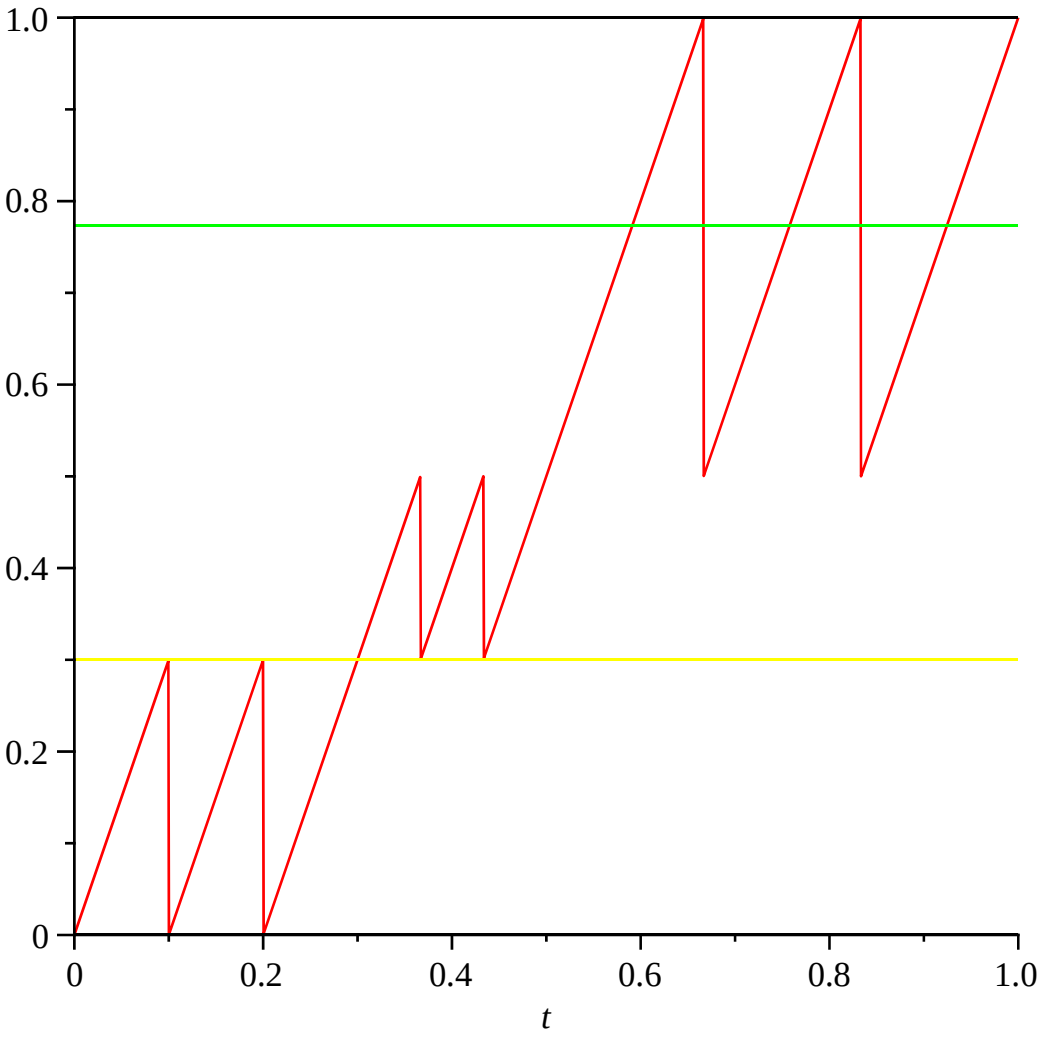
$pre_5 := 0.5243376600$

$pre_6 := 0.5910043267$

$pre_7 := 0.5910043267$

$pre_8 := 0.7576709933$

$pre_9 := 0.9243376600$



7

8

9

$err2 := 1. \cdot 10^{-100}$

>

>