Math 380 Midterm Test 27 February 1996

Professor:	*Richard Hall*
Instructions:	*Please answer both questions, which carry equal marks. Calculators are permitted.*

1. Consider the Euclidean space E^3 with its tangent spaces $T_p(E^3)$. Suppose $f = x^2z + xy^2$ is a real-valued function, $\phi = e^{xy}dx - z^2dy + (1+x)dz$ is a 1-form, $W = U_1 + xU_2 - yzU_3$ is a vector field, $\mathbf{p} = (1, -1, 2) \in E^3$, and $\mathbf{v} = (0, 1, 3)$ is a tangent vector at $\mathbf{p}$.

   (a) Find the value of $\phi(\mathbf{v})$, and also an expression for the real-valued function $\phi(W) : E^3 \to R$.

   (b) Find the 1-form $\psi = df$ and the value $df(\mathbf{v})$.

   (c) Find the wedge product $\phi \wedge d(f^2)$ and also d^2f.

   (d) Find expressions for $d\phi$ and $d^2\phi$.

   (e) Suppose that α is a curve in E^3 with $\alpha(0) = \mathbf{p}$ and $\alpha'(0) = \mathbf{v}$. Find the value of $\frac{d}{dt}W(\alpha(t))|_{t=0}$. [HINT $\nabla_{\mathbf{v}}W$]

2. Consider the curve $\alpha : I \to E^3$ given by $\alpha(t) = (t, t^2, e^t)$, where $0 \in I$.

   (a) Find the Serret-Frenet frame field $\{T, N, B\}$ at $t = 0$.

   (b) Find the curvature κ and the torsion τ at $t = 0$.

   (c) Determine whether or not α is a plane curve.

   (d) Find the centre $\mathbf{c}$ and the radius a of a circle which has the same curvature κ and tangent T as α and which lies in the same osculating plane as that of α at $t = 0$.

   (e) Explain whether or not α is a cylindrical helix and give a *rough* sketch of the curve.