CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number	Sections
MAST	**218**	**A, B**

Examination	Date	Time	Pages
Final	**December 2013**	**3 hours**	**2**

Instructors	Course Examiner
S. Twareque Ali, Ewa Duma	**S. Twareque Ali**

Instructions:

Only approved calculators permitted.

Problems of equal value. Do any 8 problems.

Show all your steps. Write complete solutions on the right hand pages of your examination booklet only.

Problem 1 : (a) Find an equation of the sphere that passes through the point $(6, -2, 3)$ and has center $(-1, 2, 1)$.

(b) Find the curve in which this sphere intersects the $yz-$plane.

(c) Find the center and radius of the sphere:

$$x^2 + y^2 + z^2 - 8x + 2y + 6z + 1 = 0$$

Problem 2 : Consider the curves γ_1 and γ_2 defined by polar equations:

$r(\theta) = 3\cos\theta$ and $r(\theta) = 1 + \cos\theta$

(a) Sketch both of them. Find the Cartesian equation of any one of them in $(x, y)-$coordinates.

(b) Find the area A of the region that lies inside γ_1 and outside γ_2.

Problem 3 : A curve is defined by the parametric equations:

$$x = \int_1^t \frac{\cos u}{u} du, \qquad y = \int_1^t \frac{\sin u}{u} du, \qquad 1 \leq t \leq 2\pi$$

(a) Find the t values where the curve has vertical tangent lines.

(b) Find the length of the curve from $t = 1$ to $t = \pi/2$.

Problem 4 : Identify and sketch the graph of the surface:

$$4x^2 + 4y^2 - 8y + z^2 = 0.$$

Problem 5 : Let $P = (1, 2, -2)$, and let L be the line given parametrically

$$L : \quad (0, 3, 1) + t\,\langle 2, -1, 3\rangle, \qquad -\infty < t < +\infty.$$

(a) Find the distance of the point P from the line L;

(b) Find the equation of the plane Q passing through point P and the line L;

Problem 6 : Let $\mathbf{r}(t),\ t \in (a, b) \in \mathbb{R}$, be a differentiable vector valued function of t.

(a) Show that

$$\frac{d}{dt}|\mathbf{r}(t)| = \frac{1}{|\mathbf{r}(t)|}\mathbf{r}(t) \cdot \mathbf{r}'(t).$$

(b) If the vector $\mathbf{r}(t)$ has constant length in the interval (a, b), show that the derivative vector is perpendicular to $\mathbf{r}(t)$ at all points of this interval.

Problem 7 : At what point on the curve $x = t^3,\ y = 3t,\ z = t^4$ is the normal plane parallel to the plane $6x + 6y - 8z - 1 = 0$?

Problem 8 : Let the position function of a particle be given by

$$\mathbf{r}(t) = t^2\ \mathbf{i} + 5t\ \mathbf{j} + +(t^2 - 16t)\ \mathbf{k}\ .$$

Compute the velocity and acceleration of the particle at any time t.
When is the speed a minimum?

Problem 9 : (a) Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the function $x^2 + 2y^2 + 3z^2 = 1$.

(b) Compute the directional derivative of the function $f(x, y, z) = xe^y + ye^z + ze^x$ at the point $(0, 0, 0)$ in the direction of the vector $\mathbf{v} = 5\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

Problem 10 : Find the absolute maximum and minimum values of the function $f(x, y) = x + y - xy$ over the closed triangular region with vertices $(0, 0),\ (0, 2)$ and $(4, 0)$.

GOOD LUCK !!!