Math 251 Sec A Midterm Test 15 October 2003

Professor:	*Richard Hall*
Instructions:	*Please answer both questions, which carry equal marks. Explain your work clearly. Calculators are permitted.*

1.

(a) Find the coordinates of the matrix $\begin{pmatrix} 0 & 2 \\ -1 & 1 \end{pmatrix}$ in the basis $\mathcal{B}$, where

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

(b) If U and W are subspaces of V whose dimension is 9, and $\dim(U) = 3$, and $\dim(W) = 5$, what could be the possible values of $\dim(U \cap W)$?

(c) If $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 22 & -20 \\ 4 & 5 & -1 \end{pmatrix}$ and B is an invertible 3×3 matrix, what is the rank of AB ?

2.

(a) If $\mathcal{A}$ is a set of vectors in a vector space V, show that $W = \mathrm{Span}(\mathcal{A})$ is a subspace of V.

(b) Consider the set of vectors in $V = \mathbb{R}^4$ given by

$$\mathcal{A} = \{(1,0,2,2), (3,0,6,0), (-1,1,0,3), (1,2,6,6)\}.$$

Find the dimension of $W = \mathrm{Span}(\mathcal{A})$, and a subset $\mathcal{B}$ of $\mathcal{A}$ which is a basis for W.